Environmental policy instruments and international rivalry

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Environmental Policy Instruments and International Rivalry

A Dynamic Analysis

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and
international rivalry:
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Proefschrift

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Contents

Chapter 1

Environmental policy competition; Introduction and review of the literature

1 Introduction

Dutch gasoline stations located at the German and Belgian borders have complained about the increase in excise duties on car fuels. This environmentally-motivated increase was introduced in July 1997. It was accompanied by a decrease in holdership taxes on cars and should contribute to a reduction in congestion and the environmental problems related to car use. The owners of gas stations at the borders claimed considerable losses, because customers could buy their fuel more cheaply at the other side of the border.

In December 1997, the parties to the 'United Nations framework on climate change' gathered in Kyoto at the third convention to discuss international action on global warming. That it is hard to reach an agreement was shown in past meetings (the Unced in Rio de Janeiro and earlier conventions in Berlin and Geneva), where results were rather limited. The reductions that have been proposed before the convention starts by three large parties, Japan, the EU and the US, differ widely.

New measures on an obligatory 'manure account,' that is, the bookkeeping of all nutrient flows in the farm, for all pig breeders are expected in the Netherlands. Protesting farmers have boycotted the rules for some time. The accounts are intended to control the flow of nutrients that pollute ground water as well as the emission of ammonia from manure. The farmers claim that the rules drive up their costs, which harms their international competitiveness.

Most environmental problems have international aspects. This is most clear for global pollution problems, such as the enhanced greenhouse effect. But also seemingly local pollution problems, such as ammonia emissions from pig breeding, may have an international dimension. Consider, for example, polluting producers who use international competition as an argument to oppose stricter environmental regulation. Or consider actual stricter environmental measures that change the production costs of internationally trade products. These changes may affect the

choices of producers of related goods in other countries and of consumers abroad.
The international dimension of environmental problems is an interesting subject of research. It is also a broad subject that has resulted in a large body of literature within the field of environmental economics. The literature can be divided roughly into two parts. One part is concerned with the most obvious international dimension of environmental problems, transboundary pollution. There are economic consequences if emissions from a polluter based in one country cause environmental damage in another country, since thus the countries' welfare becomes interdependent. The other part of the literature is concerned with the two-sided relationship between international trade and the environment. First, trade affects economic growth and economic growth, in turn, affects the emissions of pollutants. Thus trade policy influences the state of the environment in the countries involved. The other side of the relationship between trade and the environment is that environmental policy affects demand and supply functions and therefore the position of a country in the international market.
This thesis analyses mainly this latter side of the relationship: the effect of several environmental policy instruments on international trade. Only a specific type of trade is considered, namely trade by large[1] producers that act strategically on an international market. That type of trade may be considered as international competition between producers based in different countries. The phrase, 'a country's position in the international market,' then gets the more specific meaning of the competitiveness of the producers concerned. Furthermore, the thesis considers only environmental policy directed at environmental problems that arise from the use of polluting inputs in production processes. Pollution from the consumption of final goods and pollution from transport, as well as policies to diminish the related environmental damage, are not considered here.
When producers are large enough to influence market prices, strategic behaviour can be expected. The various players on the market are aware of each other's behaviour and try to influence that of the other players. Trade between a few large agents differs from trade between a large number of small agents, because of this type of strategic behaviour. When governments take interest in the competitiveness of their home firms, they also become players in this game.
With the increasing openness of economies to trade, governments are hesitant to increase the costs of domestic producers too much in comparison with producers in other countries. That tendency may lead to a too lax environmental policy from a welfare point of view. The sequel of this introductory chapter (section 2) explains under what conditions a government may deem it to be a rational strategy to distort its environmental policy in this way.
The thesis considers a country's environmental policy towards polluting producers that have international rivals. It compares environmental policy instruments, namely emission standards and emission taxes. International rivalry between producers takes the form of oligopolistic competition in the output market of these producers. Chapters 2 and 3 compare taxes and standards, when the regulator wants to attain a given environmental target and prefers high

[1] 'Large' is used in the sense that the market shares of these producers are large enough, so that they can influence market prices.

firm profits. A third instrument, emission permits, is considered in a separate chapter (chapter 4). Chapter 6 compares the distortions in taxes and standards, when the regulator balances environmental damage and firm profits. The consequences of these distortions for investment in emission reduction technology are the subject of chapter 7. As a second theme, transboundary pollution (chapter 5) and the interaction between transboundary pollution and trade (chapters 6 and 7) are analysed.

In all chapters, the firm's investment is explicitly considered to be a process that takes place over time. This distinguishes the analyses in the thesis from most other literature on environmental policy and international rivalry. Since capital accumulation is modelled as a dynamic process, it is possible to make a distinction between different strategies that a firm may use to decide on its investment rate. It is assumed that capital stocks increase gradually as a result of investment and decrease through depreciation. Different investment strategies have implications for the possibilities which the firm has to commit to certain actions through its capital stock. In turn, a firm's commitment affects how environmental policy impacts firm behaviour in equilibrium.

The next section (section 2) describes the results that have been found in the strategic trade literature under the assumption of a once-and-for-all decision on capital. Section 3 then focuses on the dynamic interactions that can be studied if differential game models are applied. Differential game models are shortly introduced and two types of decision strategies are described. Three interactions are distinguished: interactions between governments, between firms, and between firms and governments. Some results in the literature that analyses these interactions by dynamic game models are summarized. Neither of these sections is meant to be an exhaustive review of current literature. Rather, they are meant to place the analysis of the thesis into context.

2 Trade and environmental policy

Although not unhampered, economic cooperation between the countries in the European Union increases. Already this has resulted in a substantial reduction of intra-European trade barriers. Explicit trade policies are forbidden[2] within the European Union. On a global scale, agreements made in the World Trade Organization (WTO)[3] and its predecessor, the General Agreement on Tariffs and Trade (GATT) have also reduced trade barriers.[4] Furthermore, economic interdependence among nations increases. Over the period 1973-1995, the estimated value of

[2]In 1968, the then six countries of the EEC completed a customs union and abolished internal tariffs. The completion by the 15 EU countries of the single market in 1993 also reduced non-tariff-barriers such as customs control and differences in technical requirements. (See, for example, Urwin, 1995.)

[3]At the end of July 1996, 123 countries and territories were members of the WTO, which accounted for more than 90 percent of world trade (WTO, 1996).

[4]In the Uruguay Round of the GATT, the developed countries made average trade-weighted tariff cuts of 40 percent on industrial products. As a result, for developed markets as a group, the tariff average is scheduled to be reduced from 6.3 to 3.8 percent (WTO, 1996).

merchandise exports rose from 575 billion dollars to 4900 billion dollars (WTO, 1996).
When trade barriers decrease, both through deliberate trade liberalization and through technological developments that decrease transport costs, the ability for countries to independently determine their policies on numerous areas may diminish. With growing economic interdependence, policy competition may also grow.[5] International policy competition may be valued positively, when it increases the efficiency of national governments. But it may also be perceived as a danger to national welfare. Direct trade policies might be replaced with distortions in other policies that are tailored to improve the competitiveness of domestic industries. High production costs, for example due to a more stringent fiscal policy, may be perceived to set home firms at a disadvantage compared to foreign competitors. That may lead to reductions in costly social systems and environmental policies. In the extreme, the competition among countries could destroy their social systems in a 'race to the bottom.'
This danger of downward competition could also hold for environmental policy: Environmental policy measures that are directed to producers may imply an increase in their production costs. Some environmentalists therefore claim that international competition results in downward distortions of environmental policy. For them, it is a reason to oppose further reduction of trade barriers. Opponents of stricter environmental regulation use similar arguments to support their case. According to them, stricter environmental policy is unwanted, because it harms the competitiveness of home firms. The result of stricter regulation would be that foreign competitors based in countries with laxer regulation drive home firms out of the market (see Rauscher 1997, for a more extensive description of the arguments used by both groups).
The stress on international rivalry that underlies both lines of reasoning is disputed by Krugman (1996). He calls it 'Pop internationalism' and argues that countries do not compete in the same way that companies compete. Trade should not be perceived as a zero-sum game, but, rather, offers mutual gains to the countries who trade. When foreign firms are able to successfully compete with home firms, that implies that home consumers can buy cheaper products. Moreover, an increase in the income of foreign consumers implies a larger market to sell home products. Furthermore, Krugman mentions that the prosperity in a country is much more determined by internal factors than by competition from foreign countries with different cost structures. If important enough, competition from foreign countries could reduce export opportunities and lead to substitution of goods produced at home by imports. But according to Krugman, imports and exports are relatively unimportant determinants of national welfare. It should be noted that this last remark was made regarding the United States. For that country, international trade is only about 10% of GDP (Krugman, 1996). Hence, changes in export or import have relatively small effects. That is completely different for open economies like the Netherlands. In the Netherlands, trade is around 50% of GDP.[6] It seems, then, better to apply this last remark in Krugman's analysis to Europe as a whole, so that export and import to and

[5] cf. Vanheukelen, 1993, who states that the possibility for the EU countries to pursue own priorities in policy decreases, both as a consequence of restrictions from EU law and as a consequence of increased mobility of production factors.

[6] In 1996, imports had a value of 46% of GDP, while exports were 50% of GDP. (Source: CBS, Maandstatistiek van de internationale handel December 1996, electronic update June 1997 and Nationale rekeningen 1996,

from Europe are considered relatively unimportant determinants of its welfare compared to intra-European economic activities. The more important observation is, however, that general equilibrium effects matter and that countries do not compete in the same way as companies. Hence, also for individual countries within Europe that observation should be kept in mind if questions of international rivalry are considered.

The thesis focuses on models that are set up such that international rivalry matters. The functions that are applied to represent the government's objectives contain the profits of domestic firms. Moreover, the models are of a partial equilibrium nature and neglect general equilibrium effects and consumer surplus. That is done deliberately, since such a structure makes it possible to sort out the mechanisms that are at stake in a relatively simple context. To draw conclusions regarding policy in real economies, however, different models are needed, ones that pay due attention to general equilibrium effects.

The next sections review literature on the distortion of environmental policy for trade strategic reasons. First (in section 2.1) comes a brief discussion of trade under perfect competition. There follows a description of models of trade under imperfect competition (in sections 2.2 and 2.3), and thereafter (in sections 2.4 and 2.5) a discussion of the role of environmental policy in such a model of trade.

2.1 Trade under perfect competition

Since trade under perfect competition is not the subject of this thesis, this section is relatively short and far from complete. For overviews that also include an analysis of issues related to trade and environment under the assumption of perfect competition, see, for instance, Rauscher (1997, chapters 3 to 5) and Ulph (1994). In this section only comparative advantage as an explanation of trade between countries is considered. The type of trade analysed is that in final goods between countries that differ in factor endowments. The section aims to summarize the reasons for distortions in environmental policy for an open economy with perfect competition. Distortions in environmental policy have two sources: the possibility for the government to affect its country's terms of trade, and transboundary pollution.

The theory of comparative advantage dates back to Ricardo. According to this theory, trade between nations arises from differences in relative productivity. When countries specialize in the production of goods in which they have a relative comparative advantage, that is beneficial for the world as a whole, because efficiency increases. In such a world, any trade restrictions lead to sub-optimal solutions from a global point of view. If a country is not large enough to influence world market prices, it is also optimal for this individual country to set zero tariffs. Only if a country is large enough,[7] so that it can influence world market prices, it is optimal for it to set nonzero tariffs. It can improve its terms of trade when it imposes export tariffs and import duties.

Pollution from production may be considered as the use of the production factor environment.

electronic update August 1997.)

[7] Large is defined in the sense that it is producing or consuming a large part of global production.

Along that line of thought, environmental quality should be incorporated as an additional production factor into models of comparative advantage. For example, Pethig (1976) formulates variants of the factor-price equalization and comparative advantage theorem for a two-sector, two-country model with local pollution. Provided that countries apply environmental policy to restrict total pollution to a maximum level, it can be said that they have more of the factor environment, if they tolerate more pollution (have laxer environmental policy goals). High pollution tolerance may have two reasons. Either the country has low preferences for a clean environment, or its ecological system can assimilate much pollution without lasting damage to the environment. Countries that have a relative abundance of the environmental production factor should then specialize in production that is 'environmentally intensive.' This leads to the claim that it is welfare improving from a global point of view if some countries specialize in 'dirty' industries. However, as pointed out by Pethig, when countries do not apply environmental policy, while they should from a welfare point of view, these countries may lose welfare as a result of opening up to trade. That happens when they specialize in 'dirty' production and their environments deteriorate seriously. To conclude, the theorem of comparative advantage and the factor-price equalization theorem may be reformulated for the production factor environment. But, that can only be done under the assumption that the supply of this production factor is defined in accordance with a country's preferences.

Other references to analyses of trade and environmental policy under the assumption of perfect competition are, among others: Copeland (1994), Krutilla (1991) and Markusen (1975a and 1975b). Copeland considers small changes from a given second-best initial situation in environmental taxes and standards, and trade tariffs and quotas. This study examines what changes are welfare improving for a country that cannot influence market prices. He finds that results can be quite different for quantity or price policies. Krutilla analyses environmental policy, given trade policy, for a country that can influence market prices. This country should incorporate terms-of-trade effects when it decides on its optimal environmental policy. A first-best optimum is obtained if the country can adjust both tariffs and environmental taxes. If tariffs are given, then a second-best environmental tax must be computed. The first-best environmental tax must be adjusted to include terms-of-trade effects and tariff-revenue effects. The latter effect assumes that trade taxes are given and nonzero and works in a direction opposite to the terms-of-trade effect. For a country that is a net exporter, taxes should be higher to improve terms of trade. For a country that is a net importer, environmental taxes should be lower to improve terms of trade. Finally, Markusen's two papers deal with the case of transboundary pollution. Markusen (1975a) derives optimal tariffs and production taxes for a small open economy. Since pollution is a function of production of one specific good, the production tax on this good has the role of a pollution tax. The optimal non-cooperative equilibrium involves zero tariffs and a positive pollution tax, which is, however, not high enough to internalize the full external costs of emissions. A cooperative equilibrium can improve on this and will be characterized by higher pollution taxes. The first-best optimum for the world as a whole has zero tariffs and production taxes that internalize the full external costs of production. Markusen (1975b) goes into more detail on the first- and second-best tax structure for a country that can influence world prices and may choose its production and consumption taxes and its tariff structure. Second-best tax structures follow if one or more of these instruments are not available. When tariffs and consumption taxes are absent, distortions in equilibrium production taxes on the polluting good from domestic marginal external costs are due to a terms-of-trade effect, an effect on foreign production of polluting goods and an indirect effect on domestic production of changes in relative prices.

While these papers differ in focus and specific assumptions, all authors assume that producers take prices as given.[8] In a model of comparative advantage, the environment is thus considered as just a production factor, similar to labour or capital. It follows that for local pollution, under perfect competition, when trade is motivated by comparative advantage, there is no reason for governments in small open economies to distort their environmental policies for trade strategic reasons. The reason for this is straightforward. It is not optimal for a small open economy to introduce trade barriers through trade policy; consequently, neither is it optimal to distort environmental policy by making it to act as a substitute for trade policy. Formally, this is shown in Ulph (1994). He asserts that individual governments in a first-best equilibrium should set environmental policy according to the (Pigouvian) rule that marginal social damage equals marginal social costs. That is, the governments should fully internalize the externalities that are a cause of environmental problems and not introduce any distortions in environmental policy.

For a large country that can influence world market prices, the above conclusion that distortions in environmental policy are suboptimal does not hold, as was shown, for example, by Krutilla (1991). Such a country finds it optimal to set positive export taxes and import tariffs. Environmental policy distortions can act as a second-best substitute for trade policies. Translated to environmental policy, an export tax implies strict environmental policy for producers of the goods exported by the country. If a country is a net importer of some good, then a downward distortion in environmental policy (that is, a laxer environmental tax on domestic producers) would benefit the country and substitute for an import tariff (see Krutilla, 1991 and Ulph, 1994). The regulator in a country which share in world trade is large enough to give it market power, may thus have reasons to distort environmental policy (see also Markusen, 1975b, and Rauscher, 1997, chapter 5).

The theory of comparative advantage is based on the assumption that production factors are internationally immobile. It may be argued that for transboundary pollution, the environment as a production factor does not satisfy this assumption. A country that pollutes its neighbouring countries through its productive activities uses the environment of those neighbouring countries as a production factor. In that way, the intensive use of the factor environment in one country affects welfare in another country, because it diminishes the availability of this factor in the other country. That establishes a direct link between countries. As a result, the environmental policy of one country affects welfare and hence environmental policy choices in other countries[9] (cf. Markusen, 1975a). Note that this link is independent of trade. Even if both countries were

[8]Note that this does not imply that regulators take prices as given.

[9]Other factors of production may also violate the assumption of international immobility, which may have similar consequences for welfare interdependence.

Table 1: Models of environment and trade under perfect competition

Author	pollution type	trade model	subject
Pethig (1976)	local	2 sectors, 2 countries	basic trade theorems
Copeland (1994)	transboundary	n sectors, small open economy	2^{nd} best, trade & env. policies
Krutilla (1991)	transboundary	1 sector, large open ec.	$1^{st}(2^{nd})$best, trade & env. policies
Markusen (1975a)	transboundary	n sectors, small open ec.	1^{st} best, trade & env. policies
Markusen (1975b)	transboundary	2 sectors, 2 countries	1^{st}- & 2^{nd}-best, trade & env. policies
Rauscher (1997)	local & transboundary	2 goods, 2 factors, small & large open ec.	1^{st}- & 2^{nd}-best, gains from trade, trade & env. policy
Ulph (1994)	local & transboundary	1 sector, small & large open ec.	1^{st}- & 2^{nd}-best, trade & env. policy

otherwise autarchic, transboundary pollution would form a link between their economies.
When damage is only local, a national government would find it optimal to set environmental policy such that all external effects are internalized. When damage is transboundary, that is no longer the case. National governments have no immediate reason to take account of environmental damage in other countries. Free-rider incentives cause governments to set policies that are less strict than what is globally optimal and to neglect their influence on depositions in the other country (see Mäler, 1989). Transboundary pollution is hence a reason why environmental policies do not fully internalize the external effects of production, even in small countries without market power.
To conclude this section on trade under perfect competition, table 1 summarizes the articles that were described above. As long as perfect competition is maintained, in a non-cooperative equilibrium a small country cannot do better than to set environmental policy such that environmental damage at home is fully internalized (see Ulph 1994). A country with market power may want to distort environmental policy directed at pollution related to production. Transboundary pollution is another reason for (downward) distortion, which is present also if countries do not trade.

2.2 Trade under imperfect competition

The theory of comparative advantage has difficulties in explaining intra-industry trade. Intra-industry trade is the simultaneous export and import of similar products. A large and increasing share of trade consists of this type of trade. For instance, in 1990 the share of intra-industry trade in total trade for Germany ranged from 32% for fuels to 76% for chemicals.[10] Recent

[10] Source: Brakman and van Marrewijk, 1996, who cite Markusen et al., 1995.

theories of international trade, so called 'new trade theory', explain intra-industry trade. The literature contains different approaches, but most contributions characterize markets by imperfect competition. Overviews are given in Helpman and Krugman (1986) and Grossman (1992).

One part of the literature on intra-industry trade applies the model of production by monopolistic competitors. Consumers prefer a wide variety of goods, either because each individual prefers variety, or because consumers vary in their preferences. Firms specialize in the production of one variant because of scale economies. Applied to trade, this model explains intra-industry trade in differentiated products, that is, in variants of similar products. Trade, namely, enlarges the range of product-varieties available within each country. The model of trade under monopolistic competition is a general-equilibrium model. References are, for example, Helpman and Krugman (1986), Grossman and Helpman (1991). Verdier (1993) applies a model with monopolistic competition to analyse matters of environmental policy in a closed economy framework. Rauscher (1997, chapter 6) analyses environmental policy on pollution related to production, to consumption and to transport in a model of trade under monopolistic competition. He suggests that, since every producer of a variety has some monopoly power, the government may use that power and distort environmental policy upward to improve the countries' terms of trade.

A second strand of literature analyses oligopolistic competition. It studies the competition between large firms on an international market. The competitors are assumed to be situated in different countries and therefore possibly subject to different regulation. Due to the focus on the behaviour of individual firms on markets, models are of a partial equilibrium nature. Even when general equilibrium aspects are included, the analysis usually abstracts from income effects, cross-substitution effects and factor-price effects, so that it is not fit to analyse economy-wide effects.

As an answer to the question whether international rivalry may lead countries to distort their nationally preferred environmental policies, this is a promising approach. In an international oligopoly, rivalry between firms and strategic behaviour are to be expected. When the interests of specific sectors are for one reason or the other important to a government, a view on trade as international oligopolistic competition supports the adjustment of government policy directed at these sectors. One example of an adjusted policy may be environmental policy. Section 2.4 considers the application of the strategic trade model for the analysis of environmental policy in an international context. But first, the next section reviews the literature on strategic trade in general.

2.3 Trade policy competition and commitment

When international markets are characterized by perfect competition, national governments have no apparent reason to worry about the 'competitiveness' of domestic[11] firms. But in case

[11] With the possibility of foreign ownership, the question is what firms should be referred to as domestic or 'home' firms. Here, for simplicity, it is assumed that home firms are owned by inhabitants and are completely

of oligopolistic international markets, producers earn nonzero profits. If a home firm earns more profits, more earnings flow to the home country. So with imperfect competition governments have a motive (increased profits for the home firm) to care about the competitiveness of their home firms. This motive is also called rent-shifting. Other motives are the value that a government may attach to the existence of certain sectors in their country (such as an airplane-building industry) and the tax revenues that are related to the profits earned by home firms.

Brander and Spencer show in a series of articles (Spencer and Brander (1983), Brander and Spencer (1983, 1985)) that the rent-shifting motive causes nonzero tariffs to be optimal from the point of view of individual governments. A cooperative solution with zero tariffs would be better. But in the absence of cooperation, competitive behaviour between the countries results in nonzero tariffs.

Spencer and Brander (1983) establishes the role of strategic interaction in a duopoly in which firms invest first in R&D and then in output. They show that in Cournot-Nash equilibrium firms distort their R&D investment from the cost-minimizing level for strategic reasons. The reason for this is that an investment in R&D provides a quantity commitment to a firm in this setting. The investment shifts out a firm's output reaction curve, because, in Brander and Spencer's article, R&D reduces variable costs. Given the choices of the competitor, a larger level of R&D therefore implies a larger equilibrium output. In other words, investment in R&D is a commitment to a higher level of output in Brander and Spencer's model. This possibility to commit potentially improves the profit of the firm. If the other firm chooses R&D according to cost minimization, the first firm gains. But in Nash equilibrium, both firms distort their R&D and produce more output than they would do when only cost minimization motives determined R&D. As a result, the firms earn lower profits than what they could have earned if they had chosen the cost-minimizing levels of R&D. Consumers gain, because total equilibrium output increases.

The simple two-stage duopoly game explains the role of commitment devices in strategic interaction between firms. The possibility to precommit to some level of output or price can be used by a firm to shift its reaction function. As a result, the outcome of the Nash equilibrium is changed to its advantage. When, however, both players engage in such commitment, the result need not be an improvement. If the firms would refrain from strategic interaction they are able to reach a solution with higher total gains.

The importance of commitment was analysed before in a different context in the industrial organization literature. Articles by Spence (1977) and Dixit (1980) were concerned with the question how commitment could be applied by incumbents for entry deterrence. The mechanism is explained in Tirole (1988). He refers to commitment as the 'ability to burn one's bridges'. Certain actions in a preliminary stage restrict the action-set open to a player in the following stage of the game. As a result, its reaction curve shifts and the equilibrium that results is more favourable to this player.

As was described above, the commitment is undertaken by the players themselves. That is, at

located in the country. In the conclusions (chapter 8), the consequences for the results of foreign ownership are considered.

an earlier stage in the game the players realize their own commitment through some binding action. However, active government policy can also provide such commitments or add to them. The reason for the government to be interested to provide its home firm with commitments is the rent-shifting motive. Under imperfect competition with restricted entry, positive rents are earned by producers and the government is interested in shifting a larger part of these rents to home firms, through an improvement of their commitment. To analyse this motive in isolation, Brander and Spencer (1983, 1985) apply a model with two firms, one in each country, and with all consumers of the good located in third countries. If such a model is applied, the governments of the two countries whose firms produce the good are only interested in high firm profits. Note that this is a partial-equilibrium analysis: it neglects the effects of active government policy on other sectors and on entry.

In Brander and Spencer (1985), the government directly provides output commitments to firms, in the form of export subsidies. In Brander and Spencer (1983), the government gives R&D commitments through subsidies on R&D, and R&D then works as an output commitment. The 1985 paper also considers whether the analysis is robust for extension of the model with a perfectly competitive 'rest of the economy' sector and consumer preferences. In the resulting model, the non-cooperative Nash equilibrium is characterized by positive export subsidies, like in the partial-equilibrium model, but at a higher level. That is so because the partial-equilibrium model neglects consumer surplus. Since consumers gain from the increase in output that results when the firms receive export subsidies, higher subsidies result if consumer surplus is included. Apart from this extension, Brander and Spencer assume that the competitors sell all their goods to consumers in a third market. This assumption makes it possible to neglect consumer surplus. It is also important for the results that the market is integrated. That is, they assume that firms sell their product on one world market and that they cannot sell parts of their output in specific countries.

A related strand of literature is based on the different assumption that the consumers in each of the two countries form separate submarkets. Demand functions may differ in each country and the firms may price-discriminate between the two markets. These analyses explain the cross-hauling of identical goods between two countries. This type of model is hence also called a model of reciprocal demand. Markusen and Venables (1992) compare the implications for trade policy of these two different assumptions about the world market. One of their results may be roughly restated as follows: With separate submarkets, the effects of active trade policy are stronger, because in that case they only affect the market for which they are intended (cf. Markusen and Venables, 1992).

Applications to environmental policy use both types of model. For example, Kennedy (1994) is a reciprocal-demand type of model, whereas Ulph (1996b) is a Brander-Spencer type of model. The difference between the two models might be mainly a difference in focus. The reciprocal demand models usually include consumer surplus and focus on the welfare effects of policy measures. In contrast, the analyses that assume an integrated world market focus more on the rivalry between firms and the effect of government policy upon that. When the international rivalry between firms is the subject of research, the Brander-Spencer approach seems to be the elementary model. However, it should be kept in mind that consumer surplus is neglected. A collection of both types of analyses can be found in the volumes edited by Grossman (1992) and Krugman (1986). Brander (1995) is a systematic overview of oligopoly models of international rivalry and the relevant literature. Empirical applications can be found in Feenstra (1988) and in a review article by Richardson (1989).

An important conclusion from the strategic trade literature is that reasons for active trade policy exist, even in a small open economy. These reasons exist when the firms in the country compete with foreign rivals in an oligopolistic market. In that case, a government that uses the right policy can obtain welfare gains. As the empirical applications show (cf. Feenstra (1988) and Richardson (1989)), the size of these gains from active trade policy is unclear.

Two main points of critique on this type of model can be distinguished (see, for example, Dixit and Grossman, 1986).

A first problem with models of oligopolistic competition is that they often neglect general-equilibrium effects. Though Brander and Spencer (1985), for example, include some general-equilibrium effects, they still assume constant factor prices, no income effects and no substitution effects. Other authors often concentrate completely on the oligopoly in question. If not carefully interpreted, these models easily lead to the 'pop-internationalist' ideas disputed by Krugman (1996). That is, ideas that confuse the competitiveness of individual sectors with that of a whole country. It is easy to conclude from the analysis of a partial model that a particular sector should receive export subsidies for rent-shifting reasons. But in a general-equilibrium context, positive effects in one sector may be negligibly small or more than compensated by negative spillover effects in other sectors.

An example of these negative general equilibrium effects is given in Dixit and Grossman (1986). They show that when several oligopolies compete for a common resource, the favourable treatment of one sector harms profits in other sectors. The reason is that the price of the resource increases due to increased demand from the protected sector. This can be applied to environment as a resource of production, as well. A distortion that implies a laxer policy for one sector may imply that other sectors are subject to stricter environmental policies than they otherwise would be. That may happen if the regulator tries to reach a given environmental target and compensates for more pollution from the distorted sector by stricter regulation for other sectors.

Another general equilibrium aspect that is often neglected is the possibility of entry. Most analyses do not explicitly model entry. They take a small number of firms as given. In a reciprocal demand model with free entry, Markusen and Venables (1992) show that equilibrium profits for marginal firms[12] firms are zero. In the model of Markusen and Venables all firms are symmetric, so that equilibrium profits are zero for all firms. In that case, government policies that would work as a rent-shifting device no longer have any rents to shift. In a model of reciprocal demand, in which separate submarkets exist, the only effect of rent-shifting policies is then to reduce consumer surplus, because they hinder competition from foreign firms.

A second, related, criticism concerns the practical applicability of these models. Results are

[12]The 'marginal firm' decides to entry as the n^{th} firm, while for the $(n+1)^{th}$ firm, entry is not profitable.

sensitive to specific assumptions, such as the type of oligopolistic competition,[13] the size of the home market in comparison to the foreign market and the weights of firm profits, consumer surplus and other items in the welfare function. Regulators that want to apply an activist trade policy therefore need to know exactly what assumptions are relevant for the case which they want to regulate.

Dixit (1984) analyses changes in the results when the assumption of one producer in each country is altered. He considers the case in which domestic oligopolies compete with foreign oligopolies. Under Cournot competition, an active trade policy then has the usual favourable effects on competitiveness from enlarged output. But this should be balanced with an additional effect: that home oligopolists produce more in total when each of them has larger output commitments. That leads to overproduction, as seen from the point of view of a government that values home-firm profits.

Eaton and Grossman (1986) use a conjectural variations model to show how results depend on the type of oligopolistic competition. Under consistent conjectural variations, zero tariffs are optimal. Positive tariffs are optimal under conjectural variations that imply Bertrand competition, and negative tariffs are optimal under Cournot competition. Maggi (1996) analyses an alternative model that explains different types of oligopolistic competition. He considers a two-stage game, where firms first compete in capacities and then compete in prices. Rather than assuming differences in conjectural variations for the last stage, his model explains differences in type of competition from the commitment value of the capacity investments made in the first stage. It is an extension of Kreps and Scheinkman (1983), who explained Cournot competition as the result of capacity investments with complete commitment, followed by price competition. Maggi's model allows firms to produce above capacity at higher marginal costs. Similar to Eaton and Grossman, Maggi finds that optimal output subsidies (or taxes) vary with the type of competition. The stronger the commitment provided by capacity choices, the more the outcome resembles Cournot competition, the higher the optimal output subsidy.

For all these reasons, a regulator is probably never able to gather enough information to set correct trade policies. In the absence of such information, it is then better to apply zero tariffs. However, Maggi's analysis gives an exception to this conclusion. Subsidies on capacity investment are less sensitive than output subsidies to the type of competition. Even if the government does not know the strength of commitments, a small subsidy is either neutral or welfare improving.

A third point of criticism may be the prisoner dilemma that characterizes the equilibrium. That calls for careful interpretation of the results. While, given unilateral action, an active trade policy may improve the position of the country, the reaction of other countries implies that a Nash equilibrium results which may well be inferior to an equilibrium with free trade. From the existence of free-trade agreements (for example, the WTO and the EU) it seems that as far as direct trade policy is concerned, countries have found ways out of this prisoner dilemma.

Given that direct trade policies, like tariffs, are banned[14] under international free-trade agree-

[13]This refers to whether, for instance, firms compete in quantities or prices.

[14]This is obviously an exaggeration of the current situation.

ments, other policy instruments may serve as a substitute. To avoid suboptimal non-cooperative equilibria for environmental policy and other areas where policy competition can be expected (such as social security and fiscal policy), agreements similar to free-trade agreements would be needed.

However, agreement to a cooperative equilibrium is much harder in these cases for a number of reasons. First, for trade, it is clear that the cooperative equilibrium should be one of no trade barriers. For environmental policy, where the benefits are uncertain and often difficult to quantify, it is harder to determine what would be the cooperative equilibrium to agree upon. Due to differences in preferences and endowments, such an equilibrium might also entail different policies in different countries, which is second reason that makes agreements harder and non-cooperative equilibria more probable. In the absence of agreements, any policy that affects the production costs of a sector that competes internationally could in principle be (mis)used for trade strategic reasons.

In a strategic-trade model with Cournot competition, the competitiveness of a firm improves if its production costs are reduced. That may be done by lower environmental taxes, subsidies on cost-reducing R&D, export subsidies or any other policy that reduces costs. The opposite kind of measures, that lead to cost increases, would improve a firm's competitiveness under Bertrand competition. That may seem counterintuitive, but under Bertrand competition, commitment to high prices helps the firm to extract additional rents from the consumers of its product. So, in that case, consumers lose from the strategic distortion of policies for trade strategic goals.

Porter has quite another approach to competitiveness (see Porter and van der Linde, 1995). He asserts that the competitiveness of firms may improve if their costs are increased in the right way. Cost increases may force firms to improve their efficiency, which in the end improves their competitiveness. This reasoning differs from the logic of strategic trade under Bertrand competition. For now, consumers do not necessarily lose. Because firms improve their efficiency, they might be able in the end to sell their product at the same or even at a lower price than before. In environmental economics, the hypotheses thrown up by Porter have lead to many articles on this topic. Palmer, Oates and Portney (1994) give a critical discussion of the applicability of Porter's hypothesis to environmental regulation.

2.4 Environmental policy competition

This section and the next survey some papers on environmental policy competition that are related to the themes discussed in the thesis. A more complete review of the literature that analyses the possibility for distortions in environmental policy when firms are subject to international rivalry can be found in Ulph (1994). This section starts with a summary of Barrett (1994) to explain the general characteristics of a strategic trade model with environmental policy.

Barrett considers a two-stage game where, in a first stage, governments decide on environmental policy and, in a second stage, firms determine output and abatement. The strictness of environmental policy works as a commitment device, comparable to the export subsidies in Brander and Spencer (1985). In this simple two-stage setting, downward distortions in environmental policy can be explained, contrary to models of trade and environmental policy with perfect competition. The commitment effect of a downward distortion in environmental policy helps the firm to bind its hands, which benefits them in competition with the other firm. In subgame-perfect Nash equilibrium, a firm's output decisions are influenced by the environmental policy at home and in the foreign country. Barrett analyses the distortion for trade-strategic reasons of an environmental standard. Such a standard implies a commitment on the amount of polluting input used by the firm, or, alternatively on the abatement costs that it has to make. A stricter standard implies that the firm can produce fewer goods at higher costs. It has to reduce its use of the polluting input or increase its abatement activities. If the standard is set less strictly, the home firm benefits. When the firms compete in a Nash-Cournot game, a laxer environmental standard shifts out the firm's reaction function. That is, with less strict standards, for any output from the competitor, the firm will supply more. As a result, its equilibrium output and profits increase with a laxer environmental standard at home and a stricter environmental standard in the competing firm's country. However, after the discussion in section 2.3, it should not be a surprise that under Bertrand competition, the opposite results are obtained. In that case, a stricter standard (and hence a commitment to higher costs) is a competitive advantage that shifts the reaction function of the firm upward. Since the products are assumed to be less-than-perfect substitutes, the loss in market share is more than compensated by the higher price that can be asked from consumers. The equilibrium price and profits are higher, the stricter the standard is at home and the laxer the standard is in the foreign country.

Barrett concludes that his results are not robust, but case-specific. It cannot be maintained that it is always bad for a firm's international competitiveness to increase the strictness of environmental standards. This depends on the number of competing firms and on the type of competition in the output market.

Conrad (1993) considers the Nash equilibrium in environmental taxes. He obtains results comparable to Barrett's. When firms compete in a Cournot-Nash fashion, governments set taxes at lower rates than they would do under perfect competition. Conrad (1995) finds that when firms compete in Bertrand version, equilibrium taxes are higher than under perfect competition.

These analyses show that, when governments do not cooperate on environmental policy, a government that attaches value to home-firm profits as well as to a clean environment, distorts its environmental policy from the Pigouvian level under oligopolistic competition. Just as in the case of trade policies, a prisoner dilemma exists. If the government refrains from such distortions, while the government in the competing firm's country engages in them, the first country is worse off. But it is better for both countries taken together if they both set environmental policy at the Pigouvian level.

Ulph (1996b) adds a second stage to the game played by firms, in which they decide on R&D investments. The model that results is similar to Brander and Spencer (1983), which analyses R&D subsidies. In Ulph's paper, governments choose the rate at which environmental taxes should be set in equilibrium. They optimize over firm profits and environmental damage and therefore distort their taxes from the Pigouvian level. A higher environmental tax implies higher costs to the firm (as in Conrad's analysis). Governments may want to distort taxes downward to give their firms a commitment to a larger output, but have to pay for this in the form of increased environmental damage.

Although the mechanism is similar, there is also a difference with the analysis in Brander and Spencer. The R&D subsidy decreases the costs of strategic investment, the firm's own commitment device. In contrast, a distorted environmental tax is an alternative to strategic investment. In Ulph (1996b) each firm has a production function with the polluting input and capital as the only production factors. A lower environmental tax means that the polluting input, instead of investment in capital, becomes cheaper. It then depends on the specific functions and parameters of the model, whether in the distorted equilibrium (where output is higher, but the alternative production factor cheaper) strategic investment is lower or higher than without distortions. In Ulph's analysis, firms' excess investments are lower with distorted taxes than they would be when the governments set their taxes at the Pigouvian level. In contrast, R&D subsidies increase the motivation to perform excess R&D.

Firms already invest more than the cost-minimizing amount in R&D, because they use the investments to commit to higher output. With distorted environmental taxes, an additional commitment device is given to firms. This helps them to compete. For small distortions, the increase in profits more than balances the decrease in environmental quality and hence, in a Nash equilibrium, both countries distort their taxes (see Ulph, 1996b). Again, the countries would be better off if they did not act strategically and chose Pigouvian taxes. The cooperative equilibrium, where both countries and firms are assumed to cooperate, has Pigouvian taxes and no excess investments.

Ulph (1992) is concerned with the choice of policy instrument for a government that takes strategic interaction into account. This may seem to be a strange problem at first sight. Given perfect information, there seems to be no reason why policy instruments differ. But with strategic distortions of policy instruments due to imperfect competition in the output market, the effects of environmental taxes and standards differ.

The two instruments lead to different commitment possibilities for firms and therefore influence firms' strategic behaviour. Ulph analyses a model in which firms invest in R&D and face Cournot competition in a world market for output. The government uses either environmental taxes or environmental standards. These instruments are used to attain a given environmental target. Since it is assumed that output is a function of only capital and the polluting input, it follows that emission standards give firms a very strong output commitment. As a result, emission standards at home reduce the sensitivity of the home firm to foreign strategic investments. That is not the case under emission taxes, where the output commitment is smaller. Since firms have less incentive to perform excess investments under standards, and standards also result in the same environmental target as taxes, the government prefers to use environmental standards. Under such standards, the excess investment in R&D is reduced most and that is exactly the reason why it is the best choice of instrument for the government. Environmental policy instruments provide firms with an alternative commitment device for excess R&D investments. Standards work better in a Cournot oligopoly, because they give firms an output commitment, whereas taxes give firms only a price commitment to one of their production factors. In the end, the equilibrium with environmental standards in both countries has higher profits for both countries' firms than does the equilibrium with taxes[15], because excess investments are reduced.

The conclusion that standards are the Nash-equilibrium choice of policy instruments depends, however, on the assumptions about the production function in Ulph (1992). In a later paper (Ulph 1996a) that contains a model with a more general production function, Ulph shows that taxes can also be the Nash-equilibrium choice of policy instruments. With more production factors than capital and a polluting input, firms find that standards do not make them completely insensitive to strategic investments by their competitor. Then, environmental taxes may better serve their interest.

Ulph and Ulph (1996) combines the choice of policy instrument with the choice of policy level in a four-stage game. In this analysis, firms have the choice to invest either in cost reduction or in emission reduction. That leads to a variety of results. It turns out that the relation between emission and investment in emission reduction is a crucial factor.

To conclude this section, in the literature it has been shown that international rivalry under imperfect competition is a reason for distortions in environmental policy, given that trade policies are ruled out. If countries would have a full range of trade policies at their disposal, they could apply these to improve equilibrium profits for their home firms (as shown by Brander and Spencer). When these are not available due to trade agreements, environmental policy distortions are a second-best option. International rivalry then leads countries to distort their environmental policies, as a substitute for tariffs and other trade policy.

When firms also use their own commitment devices and, for example, engage in strategic investment, then distortion of environmental policy by the government reduces this strategic behaviour because it offers firms alternative commitment possibilities. How much and in what direction policy instruments are distorted depends, among other factors, on the type of conjectural variations that firms have (for example, Cournot or Bertrand competition). Table 2 summarizes the papers that were discussed in this section. These represent only a few of the large number of papers that have appeared on this subject. The effect of consumer surplus is discussed in the next section, together with other additions and adjustments.

2.5 Additions and adjustments

Transboundary pollution may strengthen distortions in environmental policy for trade-strategic reasons. Imagine a country that sets laxer standards to increase the profits of its home firm, which is in Cournot competition with a foreign firm. With purely local pollution, any reduction in the strictness of the standard results in an increase in environmental damage in the home country. But when pollution is transboundary, part of the additional emissions from laxer standards cause damage abroad rather than at home.

[15] This holds true, even if firms receive a lump-sum compensation for the taxes that they pay. Note that for the government before-tax profits are relevant, since the government can redistribute the tax revenues as lump-sum compensation.

Table 2: Environmental policy in "third country" models of trade

Author	type of model	policy instrument	competition	number	subject
Barrett (1994)	2 stages	emission standard	Cournot Bertrand	n firms	target choice
Conrad (1993)	2 stages	emission taxes	Cournot Bertrand	n firms	target choice
Ulph (1996b)	3 stages	emission taxes	Cournot	2 firms	target choice
Ulph (1992)	3 stages	taxes and standards	Cournot	2 firms	instrument choice, given target
Ulph (1996a)	3 stages	taxes and standards	Cournot	2 firms	instrument choice, given target
Ulph and Ulph (1996)	4 stages	taxes and standards	Cournot	2 firms	target choice instrument choice

Transboundary pollution also is an extra reason for governments to discourage foreign output. Decreases in the use of a polluting input by foreign firms do not affect the home country's environment in case of local pollution. With transboundary pollution, these decreases result in less damage from abroad in the home country. In that case, it is in the interest of the home country to discourage the foreign country to use polluting inputs. That could, for example, be done by discouragement of foreign output that is produced with the help of this input. To discourage foreign output, the home government could distort its environmental policy. Under Cournot competition, a home environmental policy that is less strict implies that the home firm increases its output. This diminishes the demand for output from the foreign firm. Laxer environmental policies in the home country therefore lead to lower output by the foreign firm.

Hence, transboundary pollution and trade strategic considerations are interdependent reasons to distort environmental policy from the level that is optimal from the point of view of all countries together. Rauscher (1997, chapter 6) extends Barrett's analysis to include transboundary pollution and provides a clarifying discussion of strategic trade models with environmental policy in general. He finds that transboundary pollution gives an additional distortionary effect in environmental taxes, since lower domestic taxes decrease foreign output and therefore foreign emissions that cause damage at home. Some other papers that include transboundary pollution are: Conrad (1993, 1995), Ulph and Ulph (1996), Ulph (1994), Katsoulacos et al. (1996) and Kennedy (1994).

Effects on consumer welfare were also neglected in section 2.4. Implicitly it was assumed that all buyers of the product were located in a third country. Consumers favour increased output and lower prices, in case of oligopolistic competition. Under Cournot competition, lax environmental policy increases output (that is, decreases it less than a stricter policy would do). A government that takes account of consumer surplus would set laxer environmental policy on oligopolistic producers. For the closed economy case, see Lee (1975).[16] In the international rivalry models discussed here, consumer surplus is included by Kennedy (1994) and Ulph (1996a). With Cournot competition, the downward distortions in environmental policy are reinforced when consumer surplus is included. With Bertrand competition, however, consumer interests do not coincide with firm interests. The government should balance increased home-firm profits with decreased consumer surpluses and increased environmental damage.

Different environmental policy instruments can be analysed. Most authors consider either environmental taxes or emission standards. Sartzetakis and Constantatos (1995) study tradeable emission permits. The difference between various instruments is that they provide commitment in a different way.

There is another important difference between environmental policy instruments. This is the difference in allocative efficiency, that is, in the resulting division of abatement efforts and reduction costs over firms. The bad allocative efficiency of individual standards is an important reason to prefer environmental taxes and other incentive-based instruments. Under environmental standards, firms with high and low abatement costs may be forced to the same emissions, which is not cost-effective. This is not the case under the more flexible environmental taxes. With only one firm in each country,[17] or with perfect information, so that individually tailored standards are possible, standards do not perform worse than taxes on allocative efficiency. In most models discussed here, there is one firm in each country; allocative efficiency can thus be neglected. Exceptions are the analyses by Barrett (1994) and Kennedy (1994), who also discuss an oligopoly with n firms in each country.

The contributions that were discussed until now model firms in a static and simplified way. Investment is sometimes neglected. Other papers (Ulph, (1995, 1996b)) include investment in new production processes, or investment in more environmental friendly production. These investments also serve as commitment devices for firms that are in strategic competition.

While these papers pay attention to the possibility of investment, they assume immediate adjustment of capital stocks. The two-stage model is extended with an additional stage, in which firms choose their capital stocks. No attention is paid to the inherently dynamic aspects of investment. A capital stock usually lasts longer than only one period of production, so that decisions on investment have long-run implications.

Furthermore, with more than one decision variable in a multistage model, it is just a matter of choice which decision is assumed to be taken first. There is no logical order in government policy choice and a firm's decisions on capital stocks. While it may be defended that firms decide on output and consider policies as given to them, this is less obvious when they have to make decisions on capital stocks. The assumption that policies can be reconsidered after investments have been made is a plausible one. Ulph (1995) shows that the reversal of the order of decisions can have important implications. He analyses a model in which environmental policies are determined after capital stocks have been chosen.

Part of the motivation for this thesis is to find a way to resolve the arbitrariness of multistage

[16] See also Buchanan (1969) for the optimal environmental tax on a monopolist.

[17] In that case, allocative efficiency within each country is reached per definition.

models. With differential game models it is possible to make a distinction between stock variables, which adjust only gradually, and flow variables, which adjust quickly. Furthermore, assumptions about the order in decisions can be described in a structured way by different decision strategies. The next section goes more into detail about this. It describes the basic model that is used in the thesis and reviews related literature in the field of differential games.

3 Interactions in a dynamic model of strategic trade

This section presents the strategic trade model that is used in the thesis. It distinguishes three different kinds of strategic interaction that may exist: interaction between firms, interaction between governments, and interaction between the firm and its government.

Figure 1: Interactions in a model of 2 firms and 2 governments

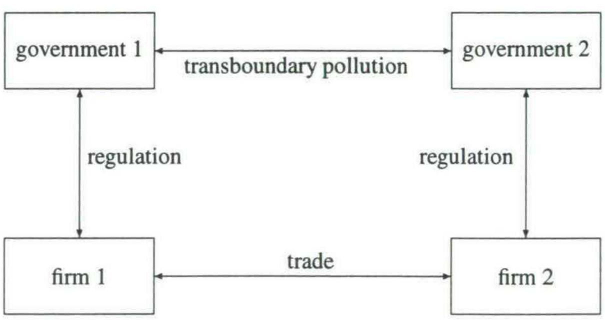


Figure 1 depicts the basic structure: Two firms in different countries compete with each other on an international market and each firm causes pollution of the own and the foreign country's environment. Thus, in each country, one specific sector is considered that is modelled as if it consists of one firm that decides on investment in a capital stock and on output. The firms are assumed to use an input that causes transboundary environmental pollution in the production process.

Governments decide on environmental policy imposed on the domestic firm, to reduce the pollution, which is caused by emissions from both firms, since pollution is transboundary. The transboundary character of pollution and interdependency on the market lead to links between the actions and objectives of all four players. If a player is aware of a link, then it may try to influence outcomes by strategic behaviour. The governments interact with each other and with the two firms. The two firms compete with each other on an international oligopolistic market for output and interact with the regulators.

The model is explicitly dynamic, that is, there is structural time dependence (cf. Friedman, 1989, p.19), because investments at some point in time t add to the accumulation of a capital stock, whose value influences profits and welfare during the whole future. Hence current pay-offs depend both on current and on past actions (on past investment decisions). Contrast this with a model that has no structural time dependence where such a stock variable is absent. That would be the case, for example, if firms would only decide on output and have no capital stock. Then there is still a possibility of time dependence in decisions, if players remember past actions of each other, but the pay-offs at some point in time do not depend on past decisions, they only depend on decisions at that time.

This structural time dependence through the accumulation of capital distinguishes the above model from the models that were discussed in the previous section. The latter models can be called 'multistage games.' They make a distinction between different stages of decision making, since players' decisions or actions may depend on actions of others, which then implicitly are assumed to be taken at an earlier stage. Hence there is some (implicit) timing of decisions involved in these models. But, these models are basically static, because all decisions are taken once and for all. There is no variable comparable to our capital stock, which accumulates over time; properly speaking, there is no time present in the model, only several stages of decision making.

As will be explained later (in section 3.1), the absence of time in these models has implications for the types of equilibria that can be analysed. A full dynamic game with structural time dependency allows for a richer set of equilibria to be formulated than does a multistage model. Some equilibria within that set have equivalents in the multistage setting. These equilibria can be seen to involve decisions that are once and for all. But other equilibria have no equivalent counterparts in a multistage model. To state the equivalence more precisely, the strategies and equilibria must be defined, which will be done in section 3.1. First, the specific model assumptions will be discussed somewhat further.

The model assumes there are two firms and two countries. This assumption is made purely to keep the model tractable. A general model would have m countries and an oligopoly consisting of m times n firms, with n firms in each country. For m countries with one firm in each country, results would not change qualitatively, but strategic effects would be smaller. The inverse demand function for output from the domestic firm would be a function of the output of the firms in all $m-1$ other countries rather than in one other country. That would imply a smaller effect of marginal changes in output of the firm in one country on the demand for output of the firm in another country than would be the case for a duopoly. With n firms in each country, another effect is added to the strategic trade effects. In that case, the home government prefers domestic firms to collude, so that they compete as if they were one firm with foreign firms. This effect works sometimes in a different direction than the effects that will be analysed in the sequel (cf. Barrett, 1994). Hence, the assumption of two countries with one firm in each country is a restrictive one, which makes strategic effects as strong as possible. Results should be interpreted keeping that in mind.

An oligopoly model with a fixed number of firms earning nonzero profits implicitly assumes that entry does not happen. A reason for the absence of entry could be that entry costs are so high that a potential entrant does not find entry profitable. In chapters 2 and 3 production functions are assumed to have increasing returns to scale which, if strong enough, could explain the absence of entry.

Competition between the firms is modelled as Cournot competition. This is only one type of oligopolistic competition that can be thought of. It should be kept in mind that results change with alternative assumptions. The choice for Cournot competition may perhaps be justified by the following remark, which is found in Helpman and Krugman (1986, p.85): 'Even though the assumption of Cournot behaviour is itself hard to justify, however, using this assumption often seems to lead to intuitively plausible conclusions. The reason for this is probably that it implies oligopolies that behave in a way intermediate between perfect competition and monopoly.'[18]

Consumer surplus is neglected. In the terminology of Brander (1995) a 'third-market model' is applied. That is, it is assumed that a large part of the consumers of the products of the two firms are located in a third country. Given Cournot competition, consumers in the two countries are interested in an increase in output (and hence in laxer environmental policies), so that the inclusion of consumer surplus would not much change the results.

The next section (section 3.1) defines decision strategies and equilibria in differential games. Section 3.2 is devoted to matters of time consistency. The three sections that follow (sections 3.3, 3.4 and 3.5) each shortly discuss dynamic-game literature related to one of the three types of interaction that can be distinguished in the basic model: interaction between firms, interaction between governments and interaction between a firm and a government.

3.1 Differential games, strategies and time paths

This thesis applies differential-game models. An important characteristic of differential game models is the presence of state variables and therefore structural time dependency. The model is called a differential game, because the development of the state variables is modelled as a system of differential equations. Similar models in discrete time, which model the development of the state variables as a system of difference equations, are called difference-game models.

In a differential game model, a player decides on intertemporal paths for the decision variables under his control. These decision variables and the decision variables controlled by other players determine the development over time of other variables, the state variables. Each player maximizes an objective function, which in its most general form is a function of the decision variables and the state variables at all points in time. With differential game models it is possible to describe the interactions between firms and governments in a structured way.[19]

Other dynamic games with or without structural time dependency can of course be formulated, for instance the difference-game models that were mentioned above. In this thesis, only

[18]Another justification for the use of Cournot competition is found in Kreps and Scheinkmann (1983), who derive Cournot competition as the result of a game of capacity competition followed by Bertrand price competition. However, that interpretation cannot be applied directly to the dynamic games analysed here, with investment in productive or abatement capital followed by output choices.

[19]For the formal definition of a differential game, see Başar and Olsder, 1995, p230.

differential game models are analysed.

The papers mentioned in section 2.3 to 2.5 have in common that they apply multistage games (most are two- or three-stage games, with governments acting in the first stage and firms in the second and third stages). That implies that decisions are assumed to take place in a fixed order and once and for all. Consequently, interactions are limited to take place in one direction only. For example, in most analyses, government decisions influence firm decisions, but not vice versa. An exception is the paper by Ulph (1995), who considers a setting in which first firms decide on investment, then governments decide on policy, and finally firms decide on output. These models are static in the sense that all decisions are made once and for all and that time plays no explicit role. But different stages of decision making are distinguished and in that sense they have some implicit dynamics. One decision is assumed to be taken before the other. This distinction of different stages explains the term multistage games. Since the stages in a multistage game take place sequentially, the game can also be seen as a difference game (see, for a formal definition, Başar and Olsder (1995, p.225)), if the state variables are properly defined.

For multistage games, several types of equilibrium can be formulated, dependent upon whether players take into account that subsequent decisions depend on past actions. If players do not act strategically (that is, if they do not take into account the effect of their decisions on subsequent decisions), then the resulting equilibria are the same as the equilibria if all players act simultaneously. If players act strategically and take the order of decisions into account, then it is appropriate to consider only subgame-perfect equilibria. These equilibria are found by going backward through the stages, using a line of reasoning analogous to Bellman's optimality principle. At a certain stage, for these equilibria, only future pay-offs resulting from an equilibrium strategy in the following stages are relevant. A definition of the concept of subgame perfection, which is due to Selten, can be found in Friedman (1989, p.81). This equilibrium concept is usually applied in multistage games.

Returning to differential game models, to describe the interactions in such a game, first a distinction is made between state and control variables. State variables cannot be controlled directly by the players. Rather, they adjust gradually as a result of the values of the control variables. One application, which is used frequently in this thesis, is to consider a firm's capital stock as a state variable that adjusts gradually according to the firm's rate of investment.

Second, the decision strategies applied by the players have to be considered. The characteristics of the decision strategy determine to what degree the actions of a player can be influenced by other players. Consider, for instance, running a race. The state of this game at some moment during the race is defined to be the position of the participating runners. This state adjusts through the speed of the runners, their control variables. A runner may choose to have the strategy: 'Run as fast as I can all the time.' Such a player's decision is not affected by any decision of the other players and neither by the state of the game. Note that the player's pay-off may be affected by the other players' decisions, though. For example, if the other players deliberately stay behind her, the result is probably different from a race in which the other players also use the strategy to run as fast as they can. Another strategy that could be chosen by a runner is this: 'Try to run at second position always.' The decision of this player, her speed, is then affected by the decisions of the other players insofar as they are reflected in their relative positions. That is, this strategy is state-dependent. When the runner with this strategy is in a first position, she must slow down. When she is in a fourth position, she should accelerate.

Formally, decision strategies are defined through the 'information sets' that players use to determine their decisions. The strategy 'run as fast as you can' does not need any information on the state of the system during the game. When such information is not available, as it would be in the hypothetical situation in which runners run in visually separated lanes, it is only possible to apply this type of strategy. With the presence of information, strategies that use the information are also possible. For example, the strategy 'run at second position' requires continuous information on the state of the system, namely on the position of the players. But only the current state is important; past positions of players are irrelevant.

Assume that the initial value of the state variables is given to the players. Then, the strategy where a player uses no information on the state variables, is called an open-loop strategy. Under such a strategy, the player chooses the time path of decisions at the start of the game and sticks to these no matter what happens. The actions of a player at some point in time depend on time only, for a given initial situation. A formal definition of an open-loop information structure is given in Başar and Olsder (1995, p.231): Denote by $\eta_i(t)$, the information set of player i at time t; then, an open-loop information structure implies that $\eta_i(t) = x_0$, the initial value of the state variables, $\forall t$. The feedback strategy is defined as a strategy, in which the decisions of a player depend only on time and the current state of the system. The information structure of a player is called a feedback information pattern if it consists of the current state. (see Başar and Olsder, 1995, p.231). Formally, the feedback information structure implies that $\eta_i(t) = x(t)$, $\forall t$, where $x(t)$ denotes the value of the state variables at time t.

If players apply feedback strategies, they react indirectly to each other's past decisions during the game, because they condition their decisions on the state of the system. In that case, a player has to take into account how her decisions influence the state of the system and hence future decisions of the other players.

Many other, more complicated strategies can be considered. For example, strategies in which the runner adjusts her speed according to the speed of some specific other runner might be thought of, or strategies that depend on all past positions of players, on the weather conditions or on the distance yet to run. Since for differential games, strategies are always defined as functions of state variables and time, not all these strategies fit into the formal framework of differential games. This thesis considers just the two types of strategies that were defined above, open-loop and feedback strategies. Since games with several players and decision variables are analysed, these two simple strategies already make it possible to analyse a variety of possible equilibria, with different assumptions about the strategies that are used for each decision variable. Furthermore, equilibria in these two types of strategies are analytically tractable, because powerful optimization techniques such as Pontryagin's maximum principle and dynamic programming can be applied.

It is important in the subsequent chapters to realize that an open-loop and a feedback strategy have different implications for players' commitments. When a player follows an open-loop strategy for a certain decision variable, it implies that she has a commitment to the equilibrium time path of the variable. She will always follow the equilibrium time path. Changes in the state of the system will not affect her decisions. If a player commits herself to a certain feedback strategy, $\gamma(x(t))$, her commitment is different. That player has no commitment to some time path of actions, for when the state of the system, $x(t)$, changes, decisions concerning a variable whose value is contingent on that state change as well. In a feedback Nash or feedback Stackelberg equilibrium, which will be defined below,[20] commitment is again different. From the definition of these equilibria it follows that there can be no commitment to either a strategy or a time path of actions. For in such an equilibrium, players reoptimize their decisions at any point in time. The value of the state variables at some point in time is, however, given to the players. So it could be said that the players are committed to these values of the state variables at some point in time, since they can only be changed gradually and not ad hoc. It can be expected that equilibria for different (combinations of) strategies have different implications for commitment and therefore for strategic behaviour and equilibrium solutions. To come back to the race example, the runner that runs as fast as she can has high commitment to do that, and nothing else. The runner that goes for a second position has no commitment to some speed, but does have a high commitment to the second position. A second competitor that overtakes her can expect that the runner will accelerate.

Given that strategies (that is, information structures) are defined and an equilibrium concept is chosen, equilibria can be formulated. In the thesis, four equilibria are considered. The open-loop Nash equilibrium is a Nash equilibrium in open-loop strategies. Its formal definition is given in Başar and Olsder (1995, theorem 6.11, p.318). The feedback Nash equilibrium is defined as the N-tuple of strategies (for a game with N players), for which value functions exist that satisfy properly stated Hamilton-Jacobi-Bellman equations for each player. The formal statement of these conditions is given in Başar and Olsder (1995, theorem 6.16, p.328).[21] The third equilibrium used in the thesis is the open-loop Stackelberg equilibrium, where the leader chooses an optimal open-loop strategy, given the equilibrium behaviour of the follower. A formal definition can again be found in Başar and Olsder (1995, p.410). The feedback Stackelberg equilibrium, finally, is defined as a Stackelberg equilibrium at any point in time t. The feedback Stackelberg equilibrium strategies define value functions that must satisfy a system of Hamilton- Jacobi-Bellman equations (see Başar and Olsder, 1995, p.418).[22] The

[20]The terminology becomes a bit confusing here. A distinction should be made between feedback strategies and feedback Nash- (or Stackelberg) equilibrium strategies. The former term refers to all strategies that depend on the current value of the state and on time, but that need not be equilibrium strategies. The latter are the equilibrium strategies in a feedback Nash equilibrium, which will be defined below.

[21]When the information structure is properly defined, this equilibrium is a special case of the wider class of so-called closed-loop no-memory Nash equilibria. Since the derivation of these more general equilibria is cumbersome, usually only feedback Nash equilibria are considered in applications. (See Başar and Olsder, 1995, section 6.5.2.)

[22]Just as is the case for the Nash equilibrium, one might think of another type of equilibrium, called the global Stackelberg equilibrium by Başar and Olsder. The definition of such an equilibrium in continuous time is, however, not so straightforward. The follower should determine its reaction to strategies that may be functions

Stackelberg leader is assumed to determine its actions before the follower at any point in time, contingent on the value of the state variables at that time. Note that these state variables change under the influence of the actions from both leader and follower. Hence, through the development of the state variables, the follower may indirectly influence the behaviour of the leader.

A third characteristic of differential game models is that they allow examination of not only steady-state equilibria, but also equilibrium paths of players' decisions during the whole period. In terms of the race example: A differential game model makes it possible to describe the characteristics of the whole race. In contrast, a static model can model only the outcome of the race. Usually however, the explicit analysis of these equilibrium paths requires very complex calculations.

After some technical questions in the next section, section 3.3 turns back to the basic model. For each of the three types of interactions (between firms, between governments and between firms and governments), a short discussion of related differential game literature is given.

3.2 Time consistency

This section addresses some technical aspects of differential games, that arise because equilibrium strategies in differential games are time paths. The definitions in Başar and Olsder (1995) are used.

A problem that needs to be considered is that an equilibrium solution at time t_0 need not be an equilibrium solution at time t_1. That is, an equilibrium need not be time consistent. When players use open-loop strategies, but they would be stopped at some point in time, told the state at that time and asked to reconsider their strategies, they might like to change the time path of their decision variable.[23] In the running example, when the value of the state variables is revealed at some point in time and a player learns that she is first, she may prefer to leave the strategy: 'run as fast as I can' and choose the strategy: 'run at a speed that I can hold on to.'

An equilibrium path $x(t)$ for a decision variable x is defined to be weakly time consistent if it is an equilibrium path from the viewpoint of any arbitrary point in time t. Formally, consider a dynamic game D, defined on the interval $[0, T]$, with strategy space Γ and solution concept *sol*.[24] Then, the following definition can be formulated:

of the development of the state variables, while the leader should optimize taking into account the reactions of the follower that may include actions that influence the development of the state variables. For a more precise explanation, see Başar and Olsder (1995).

[23] As can be seen from this description, it is a bit strange to talk about time consistency in an open-loop equilibrium, since such an equilibrium structure assumes an open-loop information pattern. With that information pattern, players get no new information after the beginning of the game. An open-loop equilibrium is very static in structure, since all decisions are 'made' at time zero. Here, the time consistency of open-loop (Nash or Stackelberg) equilibria is considered, nevertheless, in order to make some kind of comparison with feedback (Nash or Stackelberg) equilibria.

[24] This could be, for example, the (feedback or open-loop) Nash equilibrium, or the Stackelberg equilibrium.

Definition 1.1 *An N-tuple of policies $\gamma^* \in \Gamma$, solving the dynamic game $D(\Gamma;[0,T];sol)$ is weakly time consistent if its truncation to the interval $[s,T]$, $\gamma^*_{[s,T]}$, solves the truncated game $D^{\gamma^*}_{[s,T]}$, this being so for all $s \in (0,T]$. If a solution $\gamma^* \in \Gamma$ is not weakly time consistent, then it is time inconsistent.*

(quoted from Başar and Olsder, 1995, p.256). This definition is not relevant for multistage models, because the decision variables (that is the policies $\gamma^* \in \Gamma$) are not functions of time for those models. In a dynamic game, when an equilibrium strategy is not time-consistent and the players have no option to bind themselves to a time path of decisions, this implies that the equilibrium is not a credible description of what happens at future time points. If the players are able to re-optimize, they choose new equilibrium strategies, starting at that time point. It is therefore more appropriate to apply feedback (Nash or Stackelberg) equilibrium concepts in such a situation.

From the definition of a feedback Nash- and feedback Stackelberg equilibrium, it follows that policies in these equilibria are weakly time consistent. The Nash equilibrium in open-loop strategies is also a weakly time-consistent solution (see Başar and Olsder, 1995). Along the equilibrium time path there will not be a point in time when a player, being told the value of the state variables, and reconsidering her equilibrium strategy, wants to leave it. This does not hold for the open- loop Stackelberg equilibrium solution. In general, the Stackelberg equilibrium in open-loop strategies is not time consistent.

A stricter requirement is that of strong time consistency (Başar and Olsder, 1995, p.256). An equilibrium strategy is defined to be strongly time consistent if, for any value of the state variable that may occur (hence not only for values on the equilibrium path), the strategy is an equilibrium strategy. Formally, for the dynamic game $D(\Gamma;[0,T];sol)$,

Definition 1.2 *An N-tuple of policies $\gamma^* \in \Gamma$ solving the dynamic game $D(\Gamma;[0,T];sol)$ is strongly time consistent if its truncation to the interval $[s,T]$, $\gamma^*_{[s,T]}$, solves the truncated game $D^{\beta}_{[s,T]}$, for every $\beta_{[0,s)} \in \Gamma_{[0,s)}$, this being so for every $s \in (0,T]$.*

(quoted from Başar and Olsder, 1995, p.256). The feedback Nash and feedback Stackelberg equilibria are strongly time consistent, because they were defined by dynamic programming. As a consequence, equilibrium strategies in these equilibria define actions as functions of the current value of the state variables and time. Equilibria in open-loop strategies will in general not be strongly time consistent. In the open-loop Nash or open-loop Stackelberg equilibrium, actions are defined as functions of time only and dynamic programming is not used to derive the equilibrium. When, due to distortions, the state variables divert from the path they would follow in the open-loop equilibrium, then it cannot in general be expected that the open-loop equilibrium strategies are still optimal if players at some point in time learn the (distorted) value of the state variables and can reconsider their strategies.

Analyses of multistage games often consider subgame-perfect equilibria. Subgame perfection was shortly described in section 3.1. It is a concept from game-theory, which is in spirit equal to strong time consistency. But strong time consistency is defined within the structure of differential (and difference) games. In principle, a difference game (which is defined in discrete time) could be written down in its extensive form and then subgame perfection could be applied to this extensive form. For differential games, that is only possible in approximation.[25] In this thesis, strong and weak time consistency will be used for differential games and subgame perfection for multistage games. If the decisions in previous stages are defined as state variables in a difference game, then a strong time consistent equilibrium in that game is a subgame-perfect equilibrium of the multistage game.

The differential games that are analysed in this thesis all have the same, almost autonomous, structure. Let $x(t)$ denote an m-vector of state variables. And let $u(t) = (u^1(t), ..., u^i(t), ..., u^n(t))$ denote a vector of control variables, where $u^i(t)$ is the r^i-vector of controls decided on by player i. Each player i has an objective function, $J^i(x_0) = \int_0^\infty e^{-rt} g_0^i(x(t), u(t))dt$ and faces a set of constraints, divided into constraints that describe the development of the state variables, $\dot{x}(t) = g(x(t), u(t))$, and other constraints, $h(x(t), u(t)) \geq 0$. The value of the game to player i at some time t, for state variables $x(t)$, is defined by $J^i(x(t)) = \int_t^\infty e^{-rt} g_0^i(x(t), u^*(t))dt$, where the $u^*(t)$ are the time paths of decisions that are implied by equilibrium strategies. Note that time only enters explicitly in the objective function in the form of an exponential discount factor, e^{-rt}. This special structure allows for a simplification of the equilibrium conditions. It implies that equilibrium solutions do not depend on time explicitly. This simplification is used in the sequel without further notification. However, there is one exception, in the Stackelberg equilibria analysed in chapters 6 and 7: If governments apply open-loop strategies, then the objective function and constraints of firms are of a different type. For example, consider a firm that is subject to some environmental tax, τ. If $\tau(t)$ is the open-loop strategy applied by its government, this time function enters the model, so that time enters explicitly in the firm's objective function and possibly also in its constraints. Only if taxes are constant for all t does that not occurr.

Furthermore, the models in this thesis have the common feature that the time horizon is taken to be infinite. That has the implication that the value of the objective function as defined above is well-defined only for equilibria that imply time paths of decisions and state variables, whose limit, when time goes to infinity, exists and is not infinite. The simplification that time enters the objective function explicitly only through the discount factor allows for such stationary solutions. An infinite time horizon also has implications for the solution concepts that are used to derive equilibria. For open-loop (Nash or Stackelberg) equilibria, Pontryagin's Maximum Principle is applied. With an infinite time horizon, this Principle takes a slightly different form, which can be found, for instance, in Seierstad and Sydsæter (1987). For feedback (Nash or Stackelberg) equilibria, the Hamilton-Jacobi-Bellman equations are used. These equations follow directly from dynamic programming. In discrete time, for each player i, they define the value function, $J^i(x_t)$, of the game at some time t as the sum of the equilibrium pay-offs at that time and the discounted value of the game one time period further. Their continuous time variant is found by taking the limit of this expression when the length of the time period goes

[25] See Fudenberg and Tirole, 1992, for more on this issue. Their concept of Markov perfection is practically equivalent to strong time consistency.

to zero. Then the Hamilton-Jacobi-Bellman equations take a recursive form, since the value of the game at t and the value of the game at $t + \epsilon$ are equal when ϵ goes to zero. A functional equation in the value function is found, which can be used to solve for the value function, if the set-up of the game is tractable (see also de Zeeuw, 1991).

3.3 Differential game models of oligopoly

The interaction between firms in oligopolistic competition has been analysed with dynamic game models by a number of authors for the closed economy case. The most well-known type of model is probably that of a repeated one-shot oligopoly game (see, for instance, Friedman, 1989). The firms decide on one variable, output or price, in a series of repetitions of the same game. The circumstances under which they compete (for example, their production capacity) remain constant over time. So they play the same game over and over again. Therefore, this is an almost static model. There is no structural time dependency. The only dynamic aspect is that decision strategies may depend on decisions in earlier games. Considering these decision variables as state variables in a difference game in discrete time, makes it possible to apply concepts like open-loop and closed-loop strategies. These models, however, are not discussed here any further.

In the repeated game, past decisions may affect current decisions, but that is only due to assumptions about players' behaviour. Pay-offs in games at different stages do not depend on past decisions, but only on current decisions. In repeated game models of oligopoly there is no state variable that introduces structural time dependency.

Structural time dependency is implied, for example, if a state variable that represents market conditions (like the market price of the product) is included. It is then assumed that market conditions do not change abruptly, but only gradually. For example, market prices are modelled to be sticky and to adjust gradually to excess demand or supply. An oligopolist that determines his optimal output should take into account this market price.

Models of oligopoly with the market price as a state variable are Fershtman and Kamien (1987) and Tsutsui and Mino (1990). Most analyses restrict themselves to linear feedback strategies. That is, they consider only those feedback strategies in which the value of the control variables is a linear function of the state variables. This thesis also restricts the analysis of feedback strategies to linear strategies. Tsutsui and Mino (1990), however, derive equilibria in non-linear feedback strategies. Their paper shows that a continuum of equilibria in feedback strategies may exist, although the non-linear strategies are quite complicated.

When market prices are taken as the state variable, a firm's capital stock is usually considered to be given. Other dynamic oligopoly models include investment and take capital stocks as the state variables. Because each firm has its own capital stock, there will be as many state variables as players. When a duopoly is considered, that will imply two state variables. Papers that analyse this type of oligopoly games are Fershtman and de Zeeuw (1992), Flaherty (1980), Fudenberg and Tirole (1992, chapter 13), Reynolds (1987) and Stimming (1997). A difficult aspect of these models is that with two firms there are two state variables (firm one's capital stock and firm two's capital stock) and even more with more firms. Games with a relatively

simple structure, namely linear-quadratic games, already require rather complex calculations to determine equilibrium feedback investment strategies.
A feedback (Nash- or Stackelberg-) equilibrium is often a more complicated equilibrium than the open-loop equilibrium, because it enables equilibrium strategies in which firms adjust their investment to their competitor's capital stock. The investment decisions of one firm take into account that an enlarged capital stock may lead to increased (or decreased) investments from the competitor. Such interactions could lead to an 'investment race,' if an increased capital stock leads to larger competing investments. Alternatively, they could lead to a game where firms try to keep competing capital stocks low by extra investment, if an increased capital stock leads to smaller competing investments. It depends on the specific parameters of the profit function, whether firms in equilibrium invest more or less in reaction to a higher competing capital stock. In Fershtman and de Zeeuw (1992) and in Reynolds (1987), these parameters are such that higher competing capital stocks lead to lower investment. In chapter 3 a sufficient condition is derived under which a higher competing capital stock leads to lower investment.
These papers, as well as chapter 3 of this thesis, consider symmetric equilibria. Flaherty (1980) is an example of a paper that, though in a somewhat different model with open-loop investment strategies, examines asymmetric equilibria. It derives under what conditions stable asymmetric equilibria may exist. From the result of Tutsui and Mino (1990) and Flaherty (1980), it follows that the equilibria that are analysed in the thesis need not be the only equilibria existing in the game.

3.4 Dynamic models of environmental policy interaction between governments

Transboundary pollution is a source of interaction among the two governments in the basic structure (see figure 1). A special feature of environmental resources, compared to other factors of production, is that without regulation their use is free. In the absence of international agreements, a country that has its environment s polluted by foreign producers is not able to levy emission taxes on those producers or apply other environmental policy instruments. Moreover, it is hard to restrict 'migration' of environmental resources over the border. Often the substances that cross the border are diffuse and the relationship between sender and receiver is unclear.
Therefore, transboundary pollution problems require international cooperation. In a cooperative equilibrium, countries agree to adjust their environmental policy. A large literature has emerged that analyses why it is so hard to obtain an agreement. The central question in this literature is how bargaining solutions can be reached that improve on the sub-optimal Nash non-cooperative equilibrium. In that equilibrium, each government takes account of only the damage in its home country. As a result, environmental policies are distorted downward from the point of view of the world as a whole.
To concentrate on this issue, countries are usually modelled in an abstract way, with governments as the only decision makers. A distinction can be made between analyses that treat

pollution as a flow problem and those that are concerned with environmental damage related to accumulating pollution. An overview of both kind of analyses is given in Missfeldt (1996). This section restricts attention to a few papers that use a dynamic model to analyse interactions between governments when environmental damage is due to accumulated pollution. If damage is due to the accumulation of pollution, dynamic models offer a more realistic description of the decisions problems that are at stake. References that analyse models of transboundary pollution with damage from stock pollution are Hoel (1992), Dockner and Van Long (1993), Kaitala, Pohjola and Tahvonen (1992) and van der Ploeg and de Zeeuw (1992). These articles confirm the conclusion from static games of transboundary pollution (for example, Mäler 1989) that multinational cooperation improves on the non-cooperative outcome. The dynamic structure adds an incentive for distortion of individual strategies from the cooperative equilibrium (see, for example, van der Ploeg and de Zeeuw, 1992). The distortion from the global optimum is not, as in static games, due only to countries' neglect of damage across their borders. A related distortion occurs since countries want to shift costs of pollution reduction to other countries. If the state of the environment influences the decisions of the other country, a bad environment may stimulate foreign countries to higher abatement. That may lead countries to set a too lax policy, because efforts of the others offset this partially. In equilibrium, then, all countries set laxer policies than they would otherwise. To analyse this effect, a model should contain ways in which the state of the environment may influence decisions. That happens, for, example when feedback decision strategies are considered.

Without side payments, a cooperative solution is often hard to implement. For when the countries are sufficiently asymmetric, it is no sure thing, that all countries gain from an agreement. And even if all countries gain, the incentive to free ride on the cooperative behaviour of others makes a cooperative equilibrium difficult to sustain. To find equilibria that improve on the non-cooperative outcome and that can be implemented and sustained is not an easy task. Dynamic games (repeated games and differential games) make it possible to consider more complex strategies, which may be helpful in finding such equilibria.[26]

Chapter 5 in this thesis is concerned with global warming. It analyses a game between governments on emission reduction and assumes that environmental damage is caused by stock pollution. For reasons of tractability, the other chapters abstract from accumulation of pollution and assume that the government has a damage function to value damage related to flow emissions. Instead, these chapters focus on investment and have capital stocks as state variables in the game.

3.5 Dynamic models of the interaction between governments and polluters

The first chapters in this thesis (chapters 2, 3 and 4) assume that the government has a predetermined and constant emission target. If the government applies an emission tax, it is assumed that it chooses this tax such that the emission target is exactly met. However, the

[26] See for repeated games the discussion in Barrett (1996). For differential games, see chapter 4 in Cesar (1994).

firm is assumed not to act strategically with respect to this tax. That is, by assumption, the firm just takes the standard or the tax as given; no strategic interaction occurs between the firm and the government. In chapters 5, 6 and 7 this assumption is changed. Chapter 5 considers the decisions of a social planner, who determines the rate of emission through its decisions on the use of fossil fuels and the harvest rate of forests. In chapters 6 and 7, the government is assumed to be a Stackelberg leader. When it sets environmental policy according to an open-loop strategy, this implies that the firm cannot act strategically with respect to environmental policy. However, when the government is assumed to set environmental policy levels according to a feedback strategy, equilibrium environmental policy levels depend on the firm's capital stock and on the capital stock of the competitor. Since the firm realizes this, it adjusts its investment to take into account what happens to environmental policy when it increases its capital stock.

A third type of interaction to be considered, is, thus, the interaction between the firm and its regulator. A few papers that apply differential games to analyse this relationship will now be shortly discussed. Karp and Livernois (1994) is a differential game analysis that considers firms who react to given tax adjustment rules. The tax adjusts until some predetermined environmental target is reached. When firms anticipate on tax adjustments, they overreact to prevent future increases in the tax. Biglaiser, Horowitz and Quiggin (1995) assume linear damage functions and compare marketable permits and emission taxes in a closed economy model of a firm that interacts with a regulator. They derive a Stackelberg equilibrium in open-loop strategies, which need not be time consistent. In their closed economy framework, with constant marginal damage, the Pigouvian tax is constant. Therefore it is not sensitive to strategic interaction. In contrast, marketable permits are distorted due to interaction between the firm and the government. The amount of permits issued depends on the amount of investment undertaken by the firm in equilibrium.

Principal-agent interactions are an important type of interaction between the firm and its regulator. But these interactions are absent in the analysis contained in this thesis, by assumption of complete information. The regulator's uncertainty about the firm's abatement cost function and about its emissions could lead to these principal-agent interactions. The firm would have an incentive to overstate its costs to prevent stringent regulation. When the regulator has complete information about the firm, these interactions are no longer relevant.

Another assumption is that the firm has complete information about the government. Interactions related to uncertainty about future policy are therefore also neglected. See Cadot and Sinclair-Desgagné (1995) for a dynamic analysis that addresses this question.

This section concludes the description of the three types of interaction that can be distinguished. The next section finishes the introductory chapter with an overview of the contents of the chapters to follow.

4 Contents

Chapters 2 and 3 restrict attention to the interaction between firms in different countries that are subject to environmental regulation. The government has a fixed emission target and firms cannot affect their regulator. But the regulator tries to choose the environmental policy instrument that will maximize the profits of the home firm.
In chapter 2, firms are assumed to follow open-loop investment strategies and feedback output strategies. Governments have fixed constant emission targets and set their policy such that, given the state of the system, this target will be met. The chapter compares the effect on investments of emission taxes and standards. For environmental taxes, a given target implies that governments have to compute the correct tax rate in order to meet the environmental target. This tax rate thus depends on firm's capital stocks. However, firms are assumed to neglect the effect of their capital stocks on the equilibrium tax rate. For environmental standards, governments simply set the standard at the predetermined target.
Firms decide on the level of their inputs, given environmental policy. They use investments to build up a capital stock which, together with energy input, produces output. Strategic interactions between firms occur, because decisions on investments and energy use are taken consecutively. A firm's equilibrium investment strategy differs if its competitor is subject to an environmental standard or to an environmental tax. The same commitment effects that occurred in Ulph (1992) play again a role. The results in Ulph's multistage game analysis are comparable to the steady-state results in a differential game with feedback output strategies and open-loop investment strategies.
The results of chapter 2 lead to the question what happens when firms use feedback investment strategies. Chapter 3 is therefore again concerned with the interaction between two firms in a similar setting. But now the firms are assumed to follow feedback investment strategies. The game in investment that results when equilibrium output strategies are inserted is a differential game with linear dynamics, but a non-quadratic objective function. Analytically, it is hard to find an equilibrium to this game. Therefore, the equilibrium steady state is approximated with the help of an algorithm, which finds the steady state of the second-order Taylor approximation to the objective function.
It is found that, under feedback investment strategies, results can be the reverse from the conclusions that were obtained under open-loop investment strategies. This is due to the additional interactions that are added when feedback rather than open-loop investment strategies are applied.
These two chapters neglect possible transboundary aspects of pollution. The emission target is considered to be given and environmental damage is not an explicit part of the objective function. The policy instruments considered are emission taxes and emission standards.
Chapter 4 attempts to extend the analysis to emission permits. It thus develops a model of the behaviour of a firm that may invest in emission reduction and is faced with a government that issues emission permits. These emission permits may be either banked or traded, so that the firm has to decide both on its rate of investment and on the management of its permit holdings.
Chapter 5 turns to the interaction between governments when pollution is transboundary.

Specifically, this chapter considers the problem of global warming. It investigates whether equilibria change when the dynamics of carbon sequestration and emission by forests are explicitly included. This is relevant when one of the countries owns a large amount of carbon absorbing forests. The chapter analyses a model in which emissions from fossil fuel and from the use of harvested wood add to the stock of greenhouse gases in the atmosphere. Growing forests act as a sink for greenhouse gases, since they consume CO_2 in the growth.
The analysis concerns the possible conflicts and the potential cooperative gains in a two-player setting. This chapter abstracts completely from firms. As a consequence, there are no investment or output decisions to be made by individual firms or environmental policy decisions to be made by governments. Instead, the country is modelled as driven by a welfare maximizing social planner that chooses both emission rates and rates of forest harvest.
Chapters 6 and 7, attempting to combine these two aspects and include the third type of interaction, analyse a model that contains both interactions between firms, interactions between governments and interactions between the firm and its government. With transboundary pollution, governments that do not cooperate neglect the foreign effects of the emissions from their producers. Chapter 6 analyses government policies in equilibrium for three different scenarios. These three scenarions are: cooperation between firms and between governments, cooperation between governments but competition between firms, and competition between firms and governments.
Chapter 7, focusing on the effect of strategic distortions in environmental policy on the dynamic efficiency of environmental policy instruments, analyses firms' incentives to invest in emission-reducing technology under emission taxes and emission standards. Governments balance trade interests with environmental concerns. If they impose the strict policy they prefer from an environmental point of view, they might harm home firms and decrease their profits. It is assumed that governments do not want this, since they have a preference for high profits to be earned at home. This leads to downward distortions in environmental policy. These downward distortions, however, may reduce the incentive to invest in emission reduction technology. To analyse this, chapter 7 compares equilibria with and without distortions. Firms base their decisions to invest on expected future policy. Due to the distortions in environmental policy, it is not *a priori* clear which type of instrument, taxes or standards, gives the best incentive to invest in emission-reducing technologies (that is, the incentive that is closest to the social optimum). Environmental taxes are shown to be sometimes less dynamically efficient than standards.
Finally, chapter 8 summarizes the results.

Chapter 2

Standards versus taxes in a dynamic duopoly model of trade; Open-Loop investment strategies

This chapter discusses environmental policy instruments in a differential game model of international trade. The model describes a duopoly, where each competitor is situated in a different country. Emission taxes or standards are set by a fully informed regulator in such a way that a predetermined target is reached. Firms decide on the level of their inputs, given environmental policy. Investments are used to build up a capital stock which together with a polluting input produces output. Strategic interactions can occur if decisions on investments and on the polluting input are taken consecutively.
In such a setting emission taxes and standards are compared. A firm reacts differently if its competitor is subject to a standard than if it is subject to a tax. This is due to the reduction in flexibility implied by standards. In a duopoly structure this reduction in flexibility turns out to be to the advantage of the firms involved, because it reduces strategic overinvestments. The chapter discusses equilibria for the duopoly, where firms use open-loop investment strategies and use feedback strategies for their choice of the polluting input.

1 Introduction

Flexibility is an important reason to advocate economic environmental policy instruments like emission taxes. More than under traditional command and control measures, polluters can choose to what degree and in which way they decrease their pollution. Consequently, the regulator may reach a high allocative efficiency, even with inadequate information about the cost structure of the polluting firms.
In case of imperfect competition, however, flexibility has also another effect, because in such

markets commitment plays an important role (see for instance Tirole, 1988). In some cases it is advantageous for a firm in oligopoly to be able to 'burn its bridges'. The ability to bind itself to certain actions gives the firm a relatively strong position towards a competitor. More flexibility by its nature decreases the possibilities to commit.

Since an incentive for rent shifting exists in a situation of imperfect competition, governments may be interested in pursuing trade policies to increase the commitment possibilities for 'their' firms. Brander and Spencer (1983, 1985) analyse the multistage games which result when both governments and firms act strategically. It is often claimed (and disputed) that environmental policy might be (mis)used as a substitute for trade policy when the latter is prohibited in free trade agreements. Contributtions that discuss this in the context of strategic international trade were reviewed in chapter 1. Most of these papers are, in contrast with the chapter below, concerned with the level of the tax or standard applied, that is with environmental policy target setting.

In this chapter, it is assumed that the target of environmental policy is fixed beforehand to focus on a second aspect of environmental policy. When environmental policy instruments differ in the degree of commitment they confer to firms, it is of interest to single out the following question: How do environmental policy instruments differ in their effects on international trade when governments are interested in rent shifting and trade is characterized by imperfect competition? Ulph (1992) analyses a model of trade under imperfect competition in which the target of environmental policy is given. When governments apply standards rather than taxes, he finds that the reasons for firms to engage in strategic behaviour are decreased, because standards commit their competitors to certain choices. This commitment reduces the influence of strategic actions on the competitor, which actually benefits the firms.

Ulph (1992) and other papers on environmental policy and strategic trade[1] have in common that they model multistage games. Hence every decision taken is once and for all. It could be interesting to elaborate this to a fully dynamic setting for two reasons. Firstly a multistage model requires assumptions about the order in which decisions are taken. In a differential game model this order is modelled more naturally by making a distinction between state and control variables. Secondly, not only steady-state equilibria, which are comparable to the Nash equilibria in the multistage games, can be studied, but also equilibrium paths of players' decisions during the whole period. This implies that players can react to each other's past decisions during the game. Due to the fixed order of decisions and their once and for all character, in multistage models this is only possible to a certain degree: Firms can adjust output to the investment decisions taken by their competitor, but they cannot adjust decisions in reaction to the other firm's outputs. In a dynamic game environment, the impact of various assumptions about the way in which players react to each other's past decisions can be analysed.

This chapter contains an extension of Ulph's (1992) model to a fully dynamic one. The analysis focuses on the comparison of the effects of environmental policy instruments on firms. We abstract from target setting by assuming that the goal of environmental policy is fixed beforehand. A differential game model is presented with two firms that trade over an infinite

[1] These are reviewed in chapter 1.

planning period. The firms may apply either open-loop or feedback decision strategies[2]. Since firms have to decide on more than one variable, a mix of open-loop and feedback strategies is also possible. In the model presented in this chapter, firms apply such a mixed strategy. They have two decision variables: investments and the use of a polluting input, which, as an example, will often be referred to as energy in the sequel. It is assumed that the solution strategy of firms is such that they plan investment for a long period at the initial time and stick to these plans, but decide on their use of the polluting input, for instance energy, at each moment of time t, after investment has taken place. This mixed strategy is the most simple formulation that enables the study of the mechanisms described in Ulph (1992) in a dynamic game framework. We choose to stay close to Ulph's model in order to analyse the effects of a shift to a dynamic game framework as such. Chapter 3 then extends the analysis to feedback investment strategies.

Which assumption is the best to make from an economic viewpoint, depends on the specific investment and type of production that are considered. As an example of open-loop investment strategies and feedback strategies for the decision on the use of a polluting input as a reasonable assumption, think of investment as the purchase of certain production equipment. For low enough rates of discount and depreciation this decision can be modelled as taken at the start of the game. Think of the decision on energy use as the decision regarding how many hours to run the equipment per day, which is decided upon each day, depending on the situation at hand.

The model assumptions in this chapter resemble those of Ulph in so far as it is again assumed that energy use is decided upon after investment has taken place. They differ, however, because capital stock is used as a means of production during more than one period and therefore investment decisions have implications for the whole future.

Section 2 below gives a differential game model of two competing firms. In section 3 equilibrium strategies for the firms are discussed. Conclusions are drawn in section 4, which also gives some directions for further research.

2 A duopoly model

The model describes a duopoly where each competitor is situated in a different country[3] and faces the environmental policy of that country. The decision makers are firms and regulators. The regulator in each country has a fixed environmental target $\bar{e}^i(t)$ for all times t and prefers high profits for its home firm. Emissions are assumed to have local effects only[4]. Firms and countries are connected through the common market for outputs. Each firm is a profit maximizer. Throughout the whole chapter perfect foresight is assumed.

[2] See section 1.3.1 for a definition of these strategies.

[3] With equal environmental policy instruments in each country, the results on firm behaviour also apply to a duopoly in one country.

[4] Transboundary pollution is included in the analysis in chapters 6 and 7.

Environmental policy consists either of a tax or a standard on the use of the polluting input, denoted by $e^i(t)$. It is assumed that emissions are directly related to the use of this input, which could be for example energy. This implies that an emission tax or standard is equivalent to an energy tax or standard. The regulator sets the level of the tax or the standard beforehand. Taxes, τ^i, are set such that, for all t, $e^i(t) = \bar{e}^i(t)$. The regulator is assumed to have full information on the firm so that it can accomplish this. Standards are simply equal to $\bar{e}^i(t)$. Firms take the environmental policy levels as given. Under these assumptions, taxes and standards result in the same energy use.

Each firm i maximizes its discounted stream of profits $\int_0^\infty e^{-rt}\Pi^i(t)dt$, where r is a constant rate of discount. Let R^i denote revenues, x^i denote domestic and x^j foreign output, p^e the price of energy, I^i investments and $C(I^i)$ costs of investments. Then in case of a tax, firm i's profits, $\Pi^i(t)$, are given by:

$$\Pi^i(t) = R^i(x^i(t), x^j(t)) - (p^e(t) + \tau^i(t))e^i(t) - C(I^i(t)) \tag{1}$$

and in case of a standard, by:

$$\Pi^i(t) = R^i(x^i(t), x^j(t)) - p^e(t)e^i(t) - C(I^i(t)). \tag{2}$$

Here $e^i(t)$ must be smaller than or equal to $\bar{e}^i(t)$. It will be assumed that revenues, R^i, are concave in outputs (so that $\frac{\partial^2 R^i}{\partial x^{i2}} < 0$), and furthermore satisfy: $\frac{\partial^2 R^i}{\partial x^i \partial x^j} < 0$, $\frac{\partial^2 R^i}{\partial x^{j2}} = 0$ and $\frac{\partial R^i}{\partial x^j} < 0$. Output is a function of the two production factors energy and capital, $x^i = x^i(e^i, K^i)$. It is assumed that the production function exhibits increasing returns to scale with $\frac{\partial x^i}{\partial e^i} > 0$, $\frac{\partial x^i}{\partial K^i} > 0$, $\frac{\partial^2 x^i}{\partial e^{i2}} < 0$, $\frac{\partial^2 x^i}{\partial K^{i2}} < 0$. From increasing returns to scale it follows that $\frac{\partial^2 x^i}{\partial e^{i2}}\frac{\partial^2 x^i}{\partial K^{i2}} < (\frac{\partial^2 x^i}{\partial e^i \partial K^i})^2$. Furthermore $\frac{\partial^2 x^i}{\partial K^i \partial e^i} > 0$ is assumed. This means that more capital increases the marginal productivity of energy. Firms decide on energy use and investments. Output is then determined from the capital stock and energy use through the production function above.

The costs of investment, $C(I^i)$, include adjustment costs and acquisition costs. The cost function is assumed to be increasing and convex: $C'(I^i) > 0$, $C''(I^i) > 0$. Zero investment involves no costs: $C(0) = 0$. Investments add to the capital stock according to the standard equation for capital accumulation:

$$\dot{K}^i(t) = I^i(t) - \delta^i K^i(t), \tag{3}$$

where δ^i is the constant rate of depreciation.

Summarizing, the duopoly is modelled as the following differential game.

A. In case of taxes in both countries:

$$\max_{I^i \geq 0, e^i \geq 0} \int_0^\infty e^{-rt}[R^i(x^i(t), x^j(t)) - (p^e(t) + \tau^i(t))e^i(t) - C(I^i(t))]dt \tag{4}$$

$$\text{s.t. } \dot{K}^i(t) = I^i(t) - \delta^i K^i(t) \tag{5}$$

B. In case of standards in both countries:

$$\max_{I^i \geq 0, e^i \geq 0} \int_0^\infty e^{-rt}\{R^i(x^i(t), x^j(t)) - p^e(t)e^i(t) - C(I^i(t))\}dt \tag{6}$$

$$\text{s.t. } \dot{K}^i(t) = I^i(t) - \delta^i K^i(t) \tag{7}$$

$$e^i(t) \leq \bar{e}^i(t) \tag{8}$$

C. In case of a standard in country i and a tax in country j the differential game is a combination of A and B.

3 Equilibrium strategies

In order to characterize equilibria for these games, the information structure and the strategies chosen by the players need to be specified. Complete information for all players is assumed. We will assume firms' strategies to be such that they plan investment for a longer period and stick to these plans, but decide on their energy use at time t, after investment has taken place. In their investment plans firms will take into account the reaction of the competitor to their capital stock via its use of energy. In short, firms are assumed to follow open-loop strategies for investment and feedback strategies for energy use. In section 3.1 below, Nash equilibrium strategies for energy use are derived and in section 3.2 Nash equilibrium strategies for investment are considered.

3.1 Equilibrium strategies of energy use

Since energy use does not appear in the system dynamics, choices of a certain level of energy have no effect on future periods. Therefore it is possible to solve for $e^i(t)$ at each time separately as a function of $K^i(t)$, $K^j(t)$, $\tau^i(t)$ and $\tau^j(t)$ (or $\bar{e}^i(t)$ and $\bar{e}^j(t)$). Assuming that each firm takes the energy input of the other firm as given, the first-order conditions for an optimal choice of $e^1(t)$ and $e^2(t)$ can be formulated. These are the usual equilibrium conditions on marginal benefits and marginal costs. When it is assumed that in equilibrium, both firms apply positive amounts of energy, these equilibrium conditions are[5]:

A. In case of taxes in both countries:

$$\frac{\partial R^i}{\partial x^i}\frac{\partial x^i}{\partial e^i} = \tau^i + p^e, \qquad i = 1, 2. \tag{9}$$

B. In case of standards in both countries:

$$\frac{\partial R^i}{\partial x^i}\frac{\partial x^i}{\partial e^i} \geq p^e \tag{10}$$

[5]Here and in the sequel, time, t, is often suppressed to shorten notation.

$$e^i(t) \leq \bar{e}^i(t) \tag{11}$$

$$\left(\frac{\partial R^i}{\partial x^i}\frac{\partial x^i}{\partial e^i} - p^e\right)(\bar{e}^i - e^i) = 0, \qquad i = 1, 2. \tag{12}$$

C. In case of a standard in country i and a tax in country j, for country i equations (10) to (12) must be satisfied while for country j equation (9) applies.

Condition (9) states that the marginal benefit of a unit of energy (the increase in revenues due to the extra output produced with one more unit of energy) must equal the marginal costs of such an extra unit (the price of energy plus the emission tax). In case of standards it may be impossible for the firm to set marginal revenues equal to marginal costs. If the standard is binding, the only requirement is that marginal revenues are at least as high as marginal costs. In other words, it is not optimal for the firm to decrease its energy use below the standard. From (9) or (10) it follows that in the equilibrium $\frac{\partial R^i}{\partial x^i} > 0$. This is reasonable since it is never optimal for a firm to increase its output if marginal revenues are negative. Given that $\frac{\partial R^i}{\partial x^i} > 0$ it is easy to see that the second order conditions, $\frac{\partial^2 R^i}{\partial x^{i2}}(\frac{\partial x^i}{\partial e^i})^2 + \frac{\partial R^i}{\partial x^i}\frac{\partial^2 x^i}{\partial e^{i2}} < 0$, are satisfied both in case of taxes and in case of standards.

For each e^2 (or e^1) there is a unique e^{1R} (or e^{2R}) which satisfies the first order conditions, because the lefthand side of (9) and (10) is decreasing in e^i. Points (e^1, e^2) where the reaction curves $e^{1R}(e^2)$ and $e^{2R}(e^1)$ cross, are Nash equilibria. It is assumed that the amount of energy which firm i would use in a situation of monopoly, $e^{iR}(0)$, is lower than the amount necessary to drive firm j out of the market (e^i with $e^{jR}(e^i) = 0$) and vice versa. This assures an intersection of the reaction curves in the interior. A sufficient condition for the uniqueness of a Nash equilibrium is:

$$\frac{\partial^2 R^1}{\partial x^{1^2}}\frac{\partial^2 R^2}{\partial x^{2^2}} > \frac{\partial^2 R^1}{\partial x^1 \partial x^2}\frac{\partial^2 R^2}{\partial x^1 \partial x^2}\,. \tag{13}$$

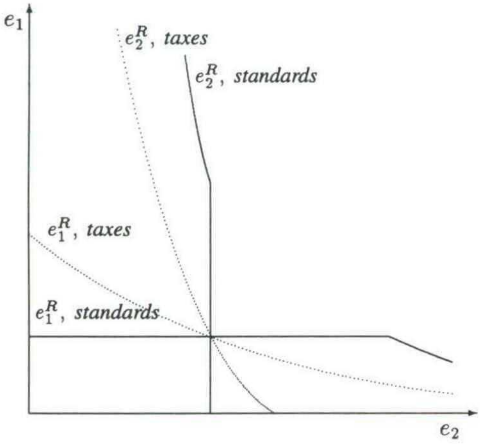
That is, the own effects on marginal revenues must dominate the cross effects. This familiar condition in duopoly models is sufficient to ensure that the Gale-Nikaido theorem can be applied to derive uniqueness of the equilibrium (Brander and Spencer, 1983). The reaction curves are drawn in figure 1 below.

From (9) and (10) to (12) the Nash equilibrium levels of energy use result in: $e^{iN}(K^1, K^2; \tau^1, \tau^2)$, $e^{iN}(K^1, K^2; \bar{e}^1, \bar{e}^2)$, or $e^{iN}(K^1, K^2; \bar{e}^i, \tau^j)$. Note that these functions are not necessarily smooth; they have kinks when $e^{iN} = \bar{e}^i(t)$ (see figure 1).

The partial derivatives $\frac{\partial e^{iN}}{\partial K^i}$ and $\frac{\partial e^{iN}}{\partial K^j}$ follow from implicit differentiation of the first order conditions. The formulas are given in appendix A. The signs of these partial derivatives are of use in what follows and are therefore given below for the cases A, B and C.

For case A, the case of taxes in both countries, $\frac{\partial e^{iN}}{\partial K^j} < 0$, while the sign of $\frac{\partial e^{iN}}{\partial K^i}$ is undetermined. This derivative splits into two parts; the first part is positive and the second part is negative. An increase in K^i increases the marginal revenue of e^i and therefore e^{iN} is expected to rise. This is captured in the first part of $\frac{\partial e^{iN}}{\partial K^i}$. But due to the substitutability of e^i and K^i, an increase in K^i also allows the same output to be produced with less e^i. This leads to a decrease in e^{iN}.

Figure 1: Reaction curves for choice of energy. (Notation: ..... = taxes in both countries, —— = standards in both countries.)



The second part of $\frac{\partial e^{iN}}{\partial K^i}$ consists of this latter effect. It is not clear beforehand which effect dominates. To get some idea about the sign of $\frac{\partial e^{iN}}{\partial K^i}$ consider the following two situations. First, let firm i be a small, growing firm with K^i small initially. Then $\frac{\partial R^i}{\partial x^i}$ will be high and presumably the first effect of increased marginal revenues will dominate. The partial derivative will be positive in this case. In the second situation, firm i is assumed to be near its 'top' level of output and $\frac{\partial R^i}{\partial x^i}$ is so low that the second effect of substitution dominates and the partial derivative is smaller than zero.

For case B, with standards in both countries, the signs depend on whether the standard is binding or not. Three possibilities exist. First, if country i's standard binds, $\frac{\partial e^{iN}}{\partial K^i} = 0$ and $\frac{\partial e^{iN}}{\partial K^j} = 0$. Second, if country i's standard does not bind, but country j's standard binds, $\frac{\partial e^{iN}}{\partial K^j} < 0$ and the sign of $\frac{\partial e^{iN}}{\partial K^j}$ is undetermined. Third, if in both countries the standard does not bind, $\frac{\partial e^{iN}}{\partial K^j} < 0$ and the sign of $\frac{\partial e^{iN}}{\partial K^i}$ is undetermined analogous to the case of taxes.

For case C, with a standard in country i and a tax in country j, the partial derivatives are zero if i's standard binds, $\frac{\partial e^{iN}}{\partial K^j} = 0$ and $\frac{\partial e^{iN}}{\partial K^i} = 0$. If i's standard does not bind, $\frac{\partial e^{iN}}{\partial K^j} < 0$ and the sign of $\frac{\partial e^{iN}}{\partial K^i}$ is undetermined. For country j, $\frac{\partial e^j}{\partial K^i} < 0$ and $\frac{\partial e^j}{\partial K^j}$ can have both signs. This case is analogous to case B with a nonbinding standard in country j.

In case of a binding standard in a country, the relevant partial derivatives are zero because it is simply forbidden for the firm in that country to increase its use of energy, while decreasing energy use is suboptimal from (10).

3.2 Equilibrium investment strategies

Investment strategies are considered to be long term decisions in this chapter. Firms plan their investments at the start ($t = 0$) and stick to these plans during the whole game. In their investment plans, they will take into account the effect of capital on energy use. Below, the first-order conditions for a Nash equilibrium of such investment strategies are derived.
The differential game can be rewritten, substituting for e^i and e^j their previously derived Nash equilibrium values:

A. In case of taxes in both countries:

$$\max_{I^i \geq 0} \int_0^\infty e^{-rt}\{R^i(x^i(t), x^j(t)) - (p^e(t) + \tau^i(t))e^{iN}(t) - C(I^i(t))\}dt \tag{14}$$

$$\text{s.t. } \dot{K}^i(t) = I^i(t) - \delta^i K^i(t), \tag{15}$$

B. In case of standards in both countries:

$$\max_{I^i \geq 0} \int_0^\infty e^{-rt}\{R^i(x^i(t), x^j(t)) - p^e(t)e^{iN}(t) - C(I^i(t))\}dt \tag{16}$$

$$\text{s.t. } \dot{K}^i(t) = I^i(t) - \delta^i K^i(t), \tag{17}$$

For case C, with a standard in country i and a tax in country j, the game is an obvious combination of A and B.
We will assume that the objective function ((14) or (16)) as a function of K^i and K^j satisfies the following regularity conditions:

$$D^i < 0 \tag{18}$$

$$E^i < 0 \tag{19}$$

$$D^1 D^2 > E^1 E^2 \tag{20}$$

Here D^i denotes the second order derivative of the objective function to own capital, $\frac{\partial^2 \Pi^i}{\partial K^{i2}}$ and E^i denotes the cross derivative of the objective function to own and foreign capital, $\frac{\partial^2 \Pi^i}{\partial K^i \partial K^j}$. The complete expressions for D^i and E^i are given in appendix B. Conditions (18) and (19) mean that the marginal profitability of own capital decreases when either own or foreign capital increases. Condition (20) means that own effects dominate cross effects. This is an analogue to assumption (13) made in section 3.1. These conditions imply a certain degree of concavity of the objective function[6].

[6]For the case of binding standards the conditions need not be assumed, since they follow from earlier assumptions on R^i and x^i.

First-order conditions for an open-loop strategy can be derived, given the competitor's investment path, $I^j(t)$, and capital stock, $K^j(t)$.

$$\lambda^i \leq C'(I^i); \;\; I^i \geq 0; \;\; I^i[C'(I^i) - \lambda^i] = 0 \tag{21}$$

$$\dot{\lambda}^i = (r + \delta^i)\lambda^i - \frac{\partial R^i}{\partial x^i}\frac{\partial x^i}{\partial K^i} - \frac{\partial R^i}{\partial x^j}\frac{\partial x^j}{\partial e^j}\frac{\partial e^{jN}}{\partial K^i} \tag{22}$$

$$\dot{K}^i = I^i - \delta^i K^i \tag{23}$$

If country j uses standards and the standards are binding, the last term in (22) disappears, because then $\frac{\partial e^{jN}}{\partial K^i}$ is zero. This is the reason for the difference between taxes and standards that occurs in this model. Standards constrain the choice set open to firms more than taxes. If the standard is binding for firm j, it will choose to use $\bar{e}^j$ units of energy and small changes in the capital stock of firm i do not influence this choice, simply because firm j cannot go beyond the standard. With taxes, firm j is more flexible in its energy use and it will react to the level of firm i's capital stock. Therefore, firm i's investment strategy will depend on the environmental policy applied in the competitor's country.
Conditions (21) to (23) can be rewritten, substituting for λ^i and $\dot{\lambda}^i$ in (22):

$$\dot{I}^i = \frac{1}{C''(I^i)}[(r + \delta^i)C'(I^i) - \frac{\partial R^i}{\partial x^i}\frac{\partial x^i}{\partial K^i} - \frac{\partial R^i}{\partial x^j}\frac{\partial x^j}{\partial e^j}\frac{\partial e^{jN}}{\partial K^i}] \tag{24}$$

$$\dot{K}^i = I^i - \delta^i K^i. \tag{25}$$

Note that only the first equation is influenced by K^j, τ^1 and τ^2 (or $\bar{e}^1$, $\bar{e}^2$). Consider first the characteristics of the steady state $(\hat{K}^i, \hat{I}^i, \hat{e}^i)$. Then $\hat{K}^i$ and $\hat{I}^i$ must satisfy:

$$\hat{I}^i = \delta^i \hat{K}^i \tag{26}$$

$$(r + \delta^i)C'(\hat{I}^i) = \frac{\partial R^i}{\partial x^i}\frac{\partial x^i}{\partial K^i} + \frac{\partial R^i}{\partial x^j}\frac{\partial x^j}{\partial e^j}\frac{\partial e^{jN}}{\partial K^i}\,. \tag{27}$$

For given $\hat{K}^j$, τ^i and τ^j (or $\bar{e}^i$ and $\bar{e}^j$), the steady state is defined by (26) and (27). A necessary condition for such a steady state to exist is that taxes or standards converge towards a constant value in the limit. Otherwise firms have to adapt their capital stocks to changing policies and a steady state cannot exist. The regularity conditions (18) and (20) are sufficient for a unique steady state to exist. From equations (26) and (27) it follows that with binding standards in country j, the steady-state capital stock in country i is lower than if taxes are applied in country j. This results from $\frac{\partial R^i}{\partial x^j}\frac{\partial x^j}{\partial e^j}\frac{\partial e^{jN}}{\partial K^i} > 0$[7] in case of taxes, while the expression equals zero in case of binding standards.

[7]From the assumptions it follows that $\frac{\partial R^i}{\partial x^j} < 0$, $\frac{\partial x^j}{\partial e^j} > 0$ and in case of taxes in country j, $\frac{\partial e^{jN}}{\partial K^i} < 0$.

The first-order conditions for an optimal path of investment, (21) and (22), can be rewritten to give a condition on $I^i(t)$ (see also Kort (1994)). Integrating (22) an expression for $\lambda^i(t)$ is obtained. Using (21) results in:

$$C'(I^i(t)) = \int_t^\infty e^{-(r+\delta^i)(s-t)}[\frac{\partial R^i}{\partial x^i}\frac{\partial x^i}{\partial K^i} + \frac{\partial R^i}{\partial x^j}\frac{\partial x^j}{\partial e^j}\frac{\partial e^j}{\partial K^i}]ds. \tag{28}$$

This condition holds along the equilibrium path, for i=1,2 and for all t. It says that the net present value of marginal investments equals zero, or in other words that marginal expenses of investment equal their discounted marginal benefits. These marginal benefits consist of two parts: direct extra revenues from increased output and indirect revenues, because firm j decreases its energy use and hence its output if firm i's capital stock increases. The indirect effect is what Ulph (1992) calls the strategic effect. Firms reason that they may gain market share by investment, because investment implies a higher capital stock, which commits them to high output in the coming periods and in this manner decreases the output profitability of the competitor. However the competitor reasons in the same way and although total output increases, the relative market shares do not change. Both total investment and output will be higher. The Nash equilibrium with strategic behaviour is then worse from the point of view of the two firms together than the Nash equilibrium that results if the firms do not act strategically. Total energy use is not affected by the firms' strategic behaviour, because environmental policy is always set such that the environmental target is met.
A path, K^{iN}, I^{iN}, where for i=1,2, it holds that:

$$C'(I^{iN}(t)) = \int_t^\infty e^{-(r+\delta^i)(s-t)}[\frac{\partial R^i}{\partial x^i}\frac{\partial x^i}{\partial K^i} + \frac{\partial R^i}{\partial x^j}\frac{\partial x^j}{\partial e^j}\frac{\partial e^j}{\partial K^i}]ds \tag{29}$$

$$\dot{K}^{iN} = I^{iN} - \delta^i K^{iN}, \tag{30}$$

is a Nash equilibrium in open-loop investment strategies. Both firms maximize their profits, given the strategy of their competitor and environmental policy. All investment decisions are taken at $t = 0$. After this time firms just follow their equilibrium investment paths. We will assume that such a Nash equilibrium exists. Given the regularity conditions (18) to (20) it can be shown that the system (21) to (23) satisfies global asymptotic stability of bounded solutions (See appendix B.). Hence, Nash-equilibrium paths will converge to the steady state. The strategic effects cause both firms to invest at a higher than optimal rate along the Nash-equilibrium time path. That can again be concluded from the positive sign of the term $\frac{\partial R^i}{\partial x^j}\frac{\partial x^j}{\partial e^j}\frac{\partial e^{jN}}{\partial K^i}$. Firms could obtain the same market shares with lower investments during the whole period if they did not try to influence each other's output (through energy use). When taxes are applied for environmental policy in both countries the equilibrium conditions are as above. In case of binding standards in both countries, the indirect effect equals zero, since firms do not change their e^i for marginal changes in K^j. In that case, condition (29) reads

$$C'(I^i(t)) = \int_t^\infty e^{-(r+\delta^i)(s-t)}[\frac{\partial R^i}{\partial x^i}\frac{\partial x^i}{\partial K^i}]ds. \tag{31}$$

It can be seen that if strategic effects are absent, the firm's capital stock is lower along the whole investment path than it would be with strategic effects.
For comparison, an equivalent to the equilibrium condition (29) is derived if the firms cooperate. An optimal investment plan results from the solution of an optimal control problem with the objective function $\int_0^\infty e^{-rt}[\Pi^i(t) + \Pi^j(t)]dt$. Following similar steps as before it is found that along an optimal investment path:

$$C'(I^{iC}(t)) = \int_t^\infty e^{-(r+\delta^i)(s-t)}[\frac{\partial R^i}{\partial x^i}\frac{\partial x^i}{\partial K^i} + \frac{\partial R^j}{\partial x^i}\frac{\partial x^i}{\partial K^i}]ds. \tag{32}$$

Now the marginal benefits of investment also include the effect of extra output on the revenues of the competitor, whereas the strategic effects are absent. It is no longer the aim of either firm to gain market share at the cost of the other. In case of cooperation, investments are lower than in the Nash equilibrium, since $\frac{\partial R^j}{\partial x^i}\frac{\partial x^i}{\partial K^i} < 0$ and $\frac{\partial R^i}{\partial x^j}\frac{\partial x^j}{\partial e^j}\frac{\partial e^{jN}}{\partial K^i} > 0$. Comparison of (29), (31) and (32) shows that standards induce firms to get closer to the cooperative solution than taxes. Under standards, less revenue can be obtained from strategic overinvestments, since both firms are committed to obeying the standards.
Summarizing, Ulph's conclusions are confirmed for an equilibrium with open-loop investment strategies in a dynamic duopoly model. Both along the investment path and in the steady state, firms have a lower capital stock in case of binding standards in both countries, than in case of taxes. This works out favourably for the firms, because they refrain from mutually harmful strategic investments.
So far investments have only been compared for the cases A and B, with the same environmental policy instrument in both countries. But a convergent Nash equilibrium also exists in case of a combination of taxes in country i and standards in country j or vice versa. The environmental policy in country j determines which net present value expression is relevant to the firm in country i.
Now it will be considered which instruments the regulators choose. Regulators were assumed to be interested in firm profits for their home firm. The same environmental target, $\bar{e}^i$, is met whether taxes or standards are applied as an instrument of environmental policy. Tax revenues flow to the regulator and can be redistributed. Hence to the regulator only profits net of taxes are of interest. If the number of home consumers is small enough compared to all consumers in the world, consumer surplus can be neglected. Given that taxes and standards result in the same environmental policy goal, which instrument will be preferred by the regulators of the two countries? To answer this question, compare the value of

$$\pi^i = \int_0^\infty e^{-rt}[R^i(x^i, x^j) - C(I^i)]dt \tag{33}$$

in the equilibria which result for various combinations of instruments in the two countries. We distinguish π^{iA}, π^{iB}, $\pi^{iC}(t,s)$ and $\pi^{iC}(s,t)$. In case C (taxes in one country and standards in the other) the first symbol in parentheses denotes the instrument applied by country i, while the second denotes the instrument in country j. In case A both countries apply taxes and in case B both countries apply standards. Note with respect to case C, that when the government in the foreign country is applying a tax, applying a standard is a dominant strategy for the home country's government. The reason is that the firm that is subject to standards may influence the behaviour of its competitor (subject to taxes), while its own energy use is bounded from above by the standard. Thus the following order of profits net of taxes can be expected:

$$\pi^{iC}(t,s) < \pi^{iA} < \pi^{iB} < \pi^{iC}(s,t)$$

It is not surprising that this again confirms the results in Ulph (1992). When taxes are used in country j, the government in country i prefers to apply standards. When standards are used in country j, the government in country i also prefers standards. It can be concluded that case B, where both regulators apply standards is the Nash equilibrium in the choice of instruments.

4 Conclusions

In a duopoly model of trade, with fixed environmental targets, emission taxes and standards do not lead to the same outcomes. Allocative efficiency plays no role in the duopoly structure, with one polluter in each country, so this is not the reason for the differences. It turns out that in this model an emission standard cures part of the inefficiencies which are due to a duopoly market structure. Strategic motives lead to investments which are inefficient from the point of view of the firms. From a social point of view the investments are also inefficient, if firm profits dominate the social objectives, given that the environmental target is reached. Hence, other things being equal, it is a Nash equilibrium if both countries choose standards as an instrument of environmental policy. However it must be kept in mind that these results depend on the specific structure of the model.

An important difference between taxes and standards as instruments of environmental policy is their allocative efficiency. Taxes result in a more efficient distribution of abatement efforts over polluters. In the model discussed above, allocative efficiency does not play a role because there is only one polluter in each country. It would be interesting to consider whether and under what circumstances the advantages of standards in coping with inefficient market structures outweigh their allocative inefficiencies.

The actual differences in profits may be quite small, because the strategic effects are second order effects. This can be seen from the formulas in appendix A. Differences in profits can be high when the market is very competitive, that is when a small decrease in the other firm's output leads to a strong increase in the market price, and when energy cannot be easily substituted, that is when firms must decrease their production if they choose to buy less energy.

Strategic investments occur in this model, because firms make short-term decisions on energy input based on the existing capital stocks. Firms increase investment, in order to decrease the profitability of energy inputs for the competitor. However, the competitor behaves similarly. A noncooperative equilibrium results, with too high investments for both firms. If they could agree to refrain from strategic investment or even to determine energy and investment in cooperation, the firms could lower their investments.

The environmental policy instruments, taxes and standards, have different implications for strategic investments. Standards for emissions which are related to energy use and hence imply energy standards reduce the range of options open to firms. If the competitor has to comply with a standard, in equilibrium, it will not adjust to small changes in the capital stock. This decreases the motives for strategic investment in the home country. Emission taxes do not change firms' flexibility and the motives for strategic investment remain. As a result, both in the steady state and in the transition to the steady state, investment is lower when standards are applied. This is consistent with the results established by Ulph (1992) for a multistage version of the model.

Investment out of strategic motives only occurs when decisions on energy use depend on the capital stocks. If firms would apply open-loop strategies both for investment and for energy use, everything would be decided at the initial time, $t = 0$, and strategic interaction during the process would disappear. In this chapter, firms apply feedback strategies for their energy use and open-loop strategies for investment. Strategies which have both investment and energy use dependent on the capital stocks are also worth consideration. This implies that firms are assumed to be more flexible in their investment decisions than before. Under that assumption firms can adjust their investment to past decisions of their rivals. A model with feedback investment strategies is analysed in chapter 3.

With feedback investment, the competitor's investments are directly influenced by the home firm's capital stock, K^i. This introduces extra strategic effects compared to open-loop investment. It is hard to conjecture as to how own capital stocks affect foreign investments. It seems that two counteracting effects are present. A higher capital stock at home reduces the revenues of the competitor and hence the profitability of its output and investments. But, a higher own capital stock might also lead the competitor to react with an increase in its own capital stock to defend its marketshare, which leads to an 'investment race'. Due to this ambiguity we need to actually compute feedback-equilibria to find the sign of $\frac{\partial I^{iN}}{\partial K^j}$.

It would be interesting to add transboundary pollution to the model. Differential game models with transboundary pollution in which governments choose their target level of environmental policy have been analysed (for example Kaitala et al., 1992; van der Ploeg and de Zeeuw, 1992). Noncooperative strategies result in a higher than optimal level of pollution, because each government neglects the effects of its emissions on foreign countries. What happens when transboundary pollution is added to the model above? To analyse this we have to change the assumptions of fixed environmental targets and introduce an explicit objective function for the government. It must make a trade-off between pollution, rents from trade and inefficiencies due to strategic firm behaviour. Chapters 6 and 7 try to answer this question and analyse a model with strategic trade and transboundary pollution.

A Partial derivatives of equilibrium emissions

This appendix gives the complete expressions for the partial derivatives mentioned in section 3.1.

A. With taxes in both countries:

$$\frac{\partial e^{iN}}{\partial K^i} =$$

$$\frac{\frac{\partial x^i}{\partial e^i}\frac{\partial x^i}{\partial K^i}[(\frac{\partial x^j}{\partial e^j})^2(\frac{\partial^2 R^i}{\partial x^i \partial x^j}\frac{\partial^2 R^j}{\partial x^i \partial x^j} - \frac{\partial^2 R^i}{\partial {x^i}^2}\frac{\partial^2 R^j}{\partial {x^j}^2}) - \frac{\partial^2 R^i}{\partial {x^i}^2}\frac{\partial R^j}{\partial x^j}\frac{\partial^2 x^j}{\partial {e^j}^2}]}{(\frac{\partial^2 R^i}{\partial {x^i}^2}(\frac{\partial x^i}{\partial e^i})^2 + \frac{\partial R^i}{\partial x^i}\frac{\partial^2 x^i}{\partial {e^i}^2})(\frac{\partial^2 R^j}{\partial {x^j}^2}(\frac{\partial x^j}{\partial e^j})^2 + \frac{\partial R^j}{\partial x^j}\frac{\partial^2 x^j}{\partial {e^j}^2}) - \frac{\partial^2 R^j}{\partial x^i \partial x^j}\frac{\partial^2 R^i}{\partial x^i \partial x^j}(\frac{\partial x^j}{\partial e^j})^2(\frac{\partial x^i}{\partial e^i})^2} -$$

$$\frac{\frac{\partial^2 x^i}{\partial e^i \partial K^i}\frac{\partial R^i}{\partial x^i}(\frac{\partial^2 R^j}{\partial {x^j}^2}(\frac{\partial x^j}{\partial e^j})^2 + \frac{\partial R^j}{\partial x^j}\frac{\partial^2 x^j}{\partial {e^j}^2})}{(\frac{\partial^2 R^i}{\partial {x^i}^2}(\frac{\partial x^i}{\partial e^i})^2 + \frac{\partial R^i}{\partial x^i}\frac{\partial^2 x^i}{\partial {e^i}^2})(\frac{\partial^2 R^j}{\partial {x^j}^2}(\frac{\partial x^j}{\partial e^j})^2 + \frac{\partial R^j}{\partial x^j}\frac{\partial^2 x^j}{\partial {e^j}^2}) - \frac{\partial^2 R^j}{\partial x^i \partial x^j}\frac{\partial^2 R^i}{\partial x^i \partial x^j}(\frac{\partial x^j}{\partial e^j})^2(\frac{\partial x^i}{\partial e^i})^2} \tag{A.1}$$

$$\frac{\partial e^{iN}}{\partial K^j} =$$

$$\frac{\frac{\partial^2 R^i}{\partial x^i \partial x^j}\frac{\partial R^j}{\partial x^j}\frac{\partial x^i}{\partial e^i}(\frac{\partial^2 x^j}{\partial e^j \partial K^j}\frac{\partial x^j}{\partial e^j} - \frac{\partial x^j}{\partial K^j}\frac{\partial^2 x^j}{\partial {e^j}^2})}{(\frac{\partial^2 R^i}{\partial {x^i}^2}(\frac{\partial x^i}{\partial e^i})^2 + \frac{\partial R^i}{\partial x^i}\frac{\partial^2 x^i}{\partial {e^i}^2})(\frac{\partial^2 R^j}{\partial {x^j}^2}(\frac{\partial x^j}{\partial e^j})^2 + \frac{\partial R^j}{\partial x^j}\frac{\partial^2 x^j}{\partial {e^j}^2}) - \frac{\partial^2 R^j}{\partial x^i \partial x^j}\frac{\partial^2 R^i}{\partial x^i \partial x^j}(\frac{\partial x^j}{\partial e^j})^2(\frac{\partial x^i}{\partial e^i})^2} < 0 \tag{A.2}$$

B. With standards in both countries:

If both standards bind, $\frac{\partial e^{iN}}{\partial K^i} = 0$; $\frac{\partial e^{iN}}{\partial K^j} = 0$, for i=1,2

If in i the standard does not bind and in j the standard binds,

$$\frac{\partial e^{iN}}{\partial K^i} = -\frac{\frac{\partial^2 R^i}{\partial {x^i}^2}\frac{\partial x^i}{\partial e^i}\frac{\partial x^i}{\partial K^i} + \frac{\partial R^i}{\partial x^i}\frac{\partial^2 x^i}{\partial e^i \partial K^i}}{\frac{\partial^2 R^i}{\partial {x^i}^2}(\frac{\partial x^i}{\partial e^i})^2 + \frac{\partial R^i}{\partial x^i}\frac{\partial^2 x^i}{\partial {e^i}^2}}; \tag{A.3}$$

$$\frac{\partial e^{iN}}{\partial K^j} = -\frac{\frac{\partial^2 R^i}{\partial x^i \partial x^j}\frac{\partial x^i}{\partial e^i}\frac{\partial x^j}{\partial K^j}}{\frac{\partial^2 R^i}{\partial {x^i}^2}(\frac{\partial x^i}{\partial e^i})^2 + \frac{\partial R^i}{\partial x^i}\frac{\partial^2 x^i}{\partial {e^i}^2}}, \tag{A.4}$$

while $\frac{\partial e^{jN}}{\partial K^i} = 0$; $\frac{\partial e^{jN}}{\partial K^j} = 0$.

If in both countries the standards do not bind, the partial derivatives are equal to those in A.

C. With a standard in i and a tax in j:

If the standard in country i binds, $\frac{\partial e^{iN}}{\partial K^i} = 0$; $\frac{\partial e^{iN}}{\partial K^j} = 0$,

while the partial derivatives for j are given by (A.3) and (A.4) with the symbols i and j interchanged. If the standard in i does not bind, the partial derivatives are equal to those in A.

B Stability conditions

This appendix gives the complete expressions for the derivatives used in the regularity conditions (18) to (20) in section 3.2 and derives a stability result. The D^i in the regularity conditions are second-order derivatives of profits with respect to own capital:

$$D^i = \frac{d^2\Pi^i}{dK^{i2}} =$$

$$\frac{\partial R^i}{\partial x^i}\frac{\partial^2 x^i}{\partial K^{i2}} + \frac{\partial^2 R^i}{\partial x^{i2}}(\frac{\partial x^i}{\partial K^i})^2 + \frac{\partial R^i}{\partial x^i}\frac{\partial^2 x^i}{\partial e^i \partial K^i}\frac{\partial e^{iN}}{\partial K^i} + \frac{\partial^2 R^i}{\partial x^{i2}}\frac{\partial x^i}{\partial K^i}\frac{\partial x^i}{\partial e^i}\frac{\partial e^{iN}}{\partial K^i} +$$

$$2\frac{\partial^2 R^i}{\partial x^i \partial x^j}\frac{\partial x^i}{\partial K^i}\frac{\partial x^j}{\partial e^j}\frac{\partial e^{jN}}{\partial K^i} + \frac{\partial R^i}{\partial x^j}\frac{\partial x^j}{\partial e^j}\frac{\partial^2 e^{jN}}{\partial K^{i2}} + \frac{\partial R^i}{\partial x^j}\frac{\partial^2 x^j}{\partial e^{j2}}(\frac{\partial e^{jN}}{\partial K^i})^2 +$$

$$\frac{\partial^2 R^i}{\partial x^{j2}}(\frac{\partial x^j}{\partial e^j})^2(\frac{\partial e^{jN}}{\partial K^i})^2 + \frac{\partial^2 R^i}{\partial x^i \partial x^j}\frac{\partial x^j}{\partial e^j}\frac{\partial e^{jN}}{\partial K^i}\frac{\partial x^i}{\partial e^i}\frac{\partial e^{iN}}{\partial K^i}. \tag{B.1}$$

The E^i are second-order cross derivatives of profits with respect to capital:

$$E^i = \frac{d^2\Pi^i}{dK^j dK^i} =$$

$$\frac{\partial^2 R^i}{\partial x^i \partial x^j}\frac{\partial x^i}{\partial K^i}\frac{\partial x^j}{\partial K^j} + \frac{\partial^2 R^i}{\partial x^i \partial x^j}\frac{\partial x^i}{\partial K^i}\frac{\partial x^j}{\partial e^j}\frac{\partial e^{jN}}{\partial K^j} + \frac{\partial^2 R^i}{\partial x^{i2}}\frac{\partial x^i}{\partial K^i}\frac{\partial x^i}{\partial e^i}\frac{\partial e^{iN}}{\partial K^j} +$$

$$\frac{\partial R^i}{\partial x^j}\frac{\partial x^j}{\partial e^j}\frac{\partial^2 e^{jN}}{\partial K^j \partial K^i} + \frac{\partial R^i}{\partial x^j}\frac{\partial^2 x^j}{\partial e^{j2}}\frac{\partial e^{jN}}{\partial K^i}\frac{\partial e^{jN}}{\partial K^j} + \frac{\partial R^i}{\partial x^j}\frac{\partial^2 x^j}{\partial e^j \partial K^j}\frac{\partial e^{jN}}{\partial K^i} +$$

$$\frac{\partial^2 R^i}{\partial x^{j2}}\frac{\partial x^j}{\partial K^j}\frac{\partial x^j}{\partial e^j}\frac{\partial e^{jN}}{\partial K^i} + \frac{\partial^2 R^i}{\partial x^{j2}}(\frac{\partial x^j}{\partial e^j})^2\frac{\partial e^{jN}}{\partial K^i}\frac{\partial e^{jN}}{\partial K^j} + \frac{\partial^2 R^i}{\partial x^i \partial x^j}\frac{\partial x^j}{\partial e^j}\frac{\partial x^i}{\partial e^i}\frac{\partial e^{jN}}{\partial K^i}\frac{\partial e^{iN}}{\partial K^j}. \tag{B.2}$$

In the following, conditions for global asymptotic stability (GAS) of the system (21) to (23) will be derived. Consider the system of four differential equations, which must hold when both firms satisfy the first-order conditions (21) to (23). When equation (21) is used to substitute for I^i in equation (23) this is a system in the variables $(K^1, \lambda^1, K^2, \lambda^2)$:

$$\dot{K}^1 = C'^{-1}(\lambda^1) - \delta^1 K^1 \tag{B.3}$$

$$\dot{\lambda}^1 = (r + \delta^1)\lambda^1 - \frac{\partial R^1}{\partial x^1}\frac{\partial x^1}{\partial K^1} - \frac{\partial R^1}{\partial x^2}\frac{\partial x^2}{\partial e^2}\frac{\partial e^{2N}}{\partial K^1} \tag{B.4}$$

$$\dot{K}^2 = C'^{-1}(\lambda^2) - \delta^2 K^2. \tag{B.5}$$

$$\dot{\lambda}^2 = (r + \delta^2)\lambda^2 - \frac{\partial R^2}{\partial x^2}\frac{\partial x^2}{\partial K^2} - \frac{\partial R^2}{\partial x^1}\frac{\partial x^1}{\partial e^1}\frac{\partial e^{1N}}{\partial K^2} \tag{B.6}$$

Sufficient conditions for this system to have a unique stationary point $(\hat{K}^1, \hat{\lambda}^1, \hat{K}^2, \hat{\lambda}^2)$ are the regularity conditions (18) and (20). Given that the system has a unique stationary point, conditions (18) and (19) are sufficient to have GAS. This follows from application of lemma 6.1 in Haurie and Leitmann (1984).

Chapter 3

Standards versus taxes in a dynamic duopoly model of trade: Feedback investment strategies.

This chapter discusses environmental policy instruments in a differential game model of international trade. The model describes a duopoly with each competitor situated in a different country. Emission taxes or standards are set by a fully informed regulator in such a way that a predetermined target is reached. Firms decide on the level of their inputs, given environmental policy. Investments are used to build up a capital stock which together with a polluting input produces output. Firms use feedback strategies for investment and therefore react on decisions of their competitor.
A firm reacts differently if its competitor is subject to an environmental standard than if it is subject to an environmental tax. As it was shown in the previous chapter, under open-loop investment strategies and feedback strategies for the polluting input, environmental taxes always give rise to more investment for strategic reasons than standards. This confirms results of multistage models of the same problem. The new result in this chapter is that under feedback investment strategies the reverse can occur.

1 Introduction

The characteristics of government policy influence the flexibility of a firm. If government policy is implemented by rigid prescriptions, it is a commitment for the firm. If, on the contrary, the government bases its policy on incentives, firms are more flexible so that they have less commitments. Brander and Spencer (1983) analysed the differences between trade policies when trade can be characterized as an international oligopoly. If governments want high profits for their home firms, they may want to provide home firms with commitment via their trade policy. If trade policies are not available, for example in case of international free trade agreements, environmental policy may be used as a substitute for an activist trade policy. Environmental policy instruments namely affect the international competitiveness of firms. It was found that environmental standards are unambiguously 'better' than taxes, in a multistage model of international rivalry where allocative efficiency can be neglected (Ulph, 1992). Ulph's analysis was extended to a fully dynamic model in the previous chapter. It was found that with open-loop investment strategies, the result carries over, that is, under standards, firms gain commitment, so that they earn more profits, while regulators reach the same environmental target.

This chapter shows that the conclusion depends on the type of investment strategy applied by firms. The analysis in chapter 2 is extended to feedback investment strategies. It will be shown that the result that standards are 'better' than taxes is ambiguous and depends on the type of investment strategy applied by the firm. To be more precise, for open-loop investment strategies the results from multistage modelling are confirmed, but for feedback investment strategies it is ambiguous whether standards or taxes are 'better'.

In a multistage model, investment decisions are taken once and for all. The implicit assumption that is implied by this type of investment decisions is that they cannot adjust, which means that they offer commitment to firms. In order to relax this assumption, the richer framework of differential games is needed. In that framework, different types of decision strategies can be distinguished.

Feedback strategies lead to more strategic interaction, which drives the firms to higher investment with lower profits. Under taxes this effect is mitigated due to the substitution between the polluting input and capital. That does not occur under standards, because in that case firms are at a corner solution for their use of the polluting input and do not substitute between that input and capital. Therefore, the use of standards reduces over-investments, which improves profits, as was found before, but it also increases over-investments, which reduces profits, since the mitigating effect is absent. The net effect can go both ways.

Section 2 presents a model of international rivalry as a differential game between two competing firms, each one situated in a different country. It is a short summary of the model, that was introduced in chapter 2. Section 3 derives the equilibrium to this model under feedback investment strategies. A comparison of the equilibrium under taxes and standards gives the main result of the chapter. The economic explanation for this result is provided at the end of this section. In section 4 we discuss choices of parameter values and how results depend on these values. Section 5 concludes the chapter.

2 An international duopoly model

The model applied in this chapter is the same as in the previous chapter. Therefore, this section only gives a quick reference of the model and the symbols used. A full description can be found in section 2.2.

Define the following list of symbols:

$x^i(t) =$ output of firm i

$I^i(t) =$ investment

$e^i(t) =$ use of energy (the polluting input)

$R^i(x^i, x^j) =$ revenues

$C(I^i) =$ investment costs

$\Pi^i(x^i, x^j) =$ firm profits

$K^i(t) =$ capital stock

$\delta^i =$ rate of depreciation

$p^e(t) =$ price of energy

$\tau^i(t) =$ environmental tax

$\bar{e}^i(t) =$ environmental standard

$r =$ discount rate.

The duopoly is modelled as the following differential game[1]:

A. In case of taxes in both countries

$$\max_{I^i \geq 0, e^i \geq 0} \int_0^\infty e^{-rt}[R^i(x^i, x^j) - (p^e + \tau^i)e^i - C(I^i)]dt \tag{1}$$

$$\text{s.t. } \dot{K}^i = I^i - \delta^i K^i \tag{2}$$

B. In case of standards in both countries

$$\max_{I^i \geq 0, e^i \geq 0} \int_0^\infty e^{-rt}[R^i(x^i, x^j) - p^e e^i - C(I^i)]dt \tag{3}$$

$$\text{s.t. } \dot{K}^i = I^i - \delta^i K^i \tag{4}$$

$$e^i \leq \bar{e}^i \tag{5}$$

C. In case of a standard in country i and a tax in country j the differential game is a combination of A and B.

[1] If not confusing, time, t, is suppressed to shorten notation

3 Equilibrium conditions and economic interpretation

3.1 Equilibrium strategies of energy use

Since energy use does not appear in the system dynamics, choices of a certain level of energy have no effect on future periods. Therefore it is possible to solve for the e^i at each time separately as a function of K^i, K^j, τ^i and τ^j (or $\bar{e}^i$ and $\bar{e}^j$). Assuming that each firm takes the energy input of the other firm as given, the first-order conditions for an optimal choice of e^1 and e^2 can be formulated. These are the usual equilibrium conditions on marginal benefits and marginal costs:

A. In case of taxes in both countries

$$\frac{\partial R^i}{\partial x^i}\frac{\partial x^i}{\partial e^i} = \tau^i + p^e, \qquad i = 1, 2. \tag{6}$$

B. In case of standards in both countries

$$\frac{\partial R^i}{\partial x^i}\frac{\partial x^i}{\partial e^i} \geq p^e \tag{7}$$

$$e^i \leq \bar{e}^i \tag{8}$$

$$(\frac{\partial R^i}{\partial x^i}\frac{\partial x^i}{\partial e^i} - p^e)(\bar{e}^i - e^i) = 0, \qquad i = 1, 2. \tag{9}$$

C. In case of a standard in country i and a tax in country j, for country i equations (7) to (9) must be satisfied while for country j equation (6) applies.

An analysis of the Nash-equilibrium strategies of energy use, $e^{iN}(K^i, K^j)$, implied by these conditions is given in chapter 2 (section 3.1). If the two firms cooperate to maximize joint profits, the first-order conditions for energy use in case of taxes in both countries are:

$$\frac{\partial R^i}{\partial x^i}\frac{\partial x^i}{\partial e^i} + \frac{\partial R^j}{\partial x^i}\frac{\partial x^i}{\partial e^i} = \tau^i + p^e, \qquad i = 1, 2. \tag{10}$$

Compare this condition to condition (6). With cooperation, effects on foreign profits are included in the marginal benefits. Analogously, in case of standards, first-order conditions for cooperation are modifications of (7) to (9). The resulting optimal levels of energy use are denoted by $e^{iC}(K^i, K^j)$.

3.2 Equilibrium strategies of investment

In this model, firms are in a dynamic environment where they have to make decisions over a longer period of time. Different strategies can be distinguished. If firms apply open-loop strategies, they do not react on current state variables. Open-loop investment strategies are a

function of time only, $I^{iOL}(t)$. If players apply feedback strategies, they react indirectly to each other's past decisions, as far as these are reflected in the current value of the state variables (K^1 and K^2). Therefore, each firm must take into account how its decisions will influence the state of the system and hence future decisions of the competitor. Feedback investment strategies are a function of time and the capital stocks, $I^{iFB}(t, K^1, K^2)$.
It is not clear ex ante how firms will react to a higher competing capital stock, that is whether the derivative $\frac{\partial I^{iFB}}{\partial K^j}$ is positive or negative. The effect of the competitor's capital stock on investment can be negative for the reason that a higher capital stock of the competitor implies that the competitor produces more output. This decreases the profitability of output and investment to the home firm. The effect can be positive, though, for the reason that a higher competing capital stock induces the firm to increase its investments to keep its market share.
The equilibrium under environmental taxes and standards with open-loop investment strategies was analysed in the previous chapter. To compute an equilibrium of feedback strategies for the general formulation of the differential game in section 2 is difficult. But it is possible to approximate the steady-state capital stock in the feedback equilibrium for explicit functional forms. Therefore consider the following scenario:

- Prices of output follow from market equilibrium on a world market with a linear inverse demand curve, $p = p_0 - x^i - \alpha x^j$. The parameter α, $0 \leq \alpha \leq 1$, denotes the degree of substitutability between the products. Gross revenues, R^i, are given by $R^i = px^i = p_0 x^i - (x^i)^2 - \alpha x^i x^j$.
- Technology is characterized by the Cobb-Douglas production function $x^i = \sqrt{e^i} K^{i\beta}$.
- Investment costs are quadratic, $C(I^i) = \frac{1}{2} c I^{i2}$.

Furthermore, it is assumed that the two countries are symmetric, that is, they are assumed to have equal rates of depreciation and discount, δ and r, equal production functions and the same, constant, targets of environmental policy, $\bar{e}$. Energy prices, p^e, are assumed to be constant. The technology was assumed to be increasing returns to scale[2], so that $\beta > \frac{1}{2}$. Marginal productivity of capital is decreasing, so that $\beta < 1$. It is straightforward to check that these functional forms satisfy the assumptions made in section 2.2.
With these functional forms the expression for the Nash equilibrium value of energy use in section 3.1 in case of taxes in both countries becomes:

$$e^{iN}(K^i, K^j) = \frac{p_0^2 K^{i2\beta}(2(p^e + \tau^j) + (2-\alpha)K^{j2\beta})^2}{(4(p^e + \tau^i + K^{i2\beta})(p^e + \tau^j + K^{j2\beta}) - a^2 K^{i2\beta} K^{j2\beta})^2}. \tag{11}$$

Insert the functions in the differential game to obtain:

$$\max_{I^i \geq 0} \int_0^\infty e^{-rt} \Pi^i(K^i, K^j) dt \tag{12}$$

$$\text{s.t. } \dot{K}^i = I^i - \delta K^i \quad i = 1, 2 \tag{13}$$

[2]See the assumptions made in section 2.2.

where

$$\Pi^i(K^i, K^j) = \frac{p_0^2 K^{i2\beta}(2(p^e+\tau^j)+(2-\alpha)K^{j2\beta})^2(p^e+\tau^i+K^{i2\beta})}{(4(p^e+\tau^i+K^{i2\beta})(p^e+\tau^j+K^{j2\beta})-a^2K^{i2\beta}K^{j2\beta})^2} - \frac{1}{2}cI^{i2}. \tag{14}$$

In case of standards in both countries, $e^i = \bar{e}^i$ and

$$\Pi^i(K^i, K^j) = p_0\sqrt{\bar{e}^i}K^{i\beta} - \bar{e}^i K^{i2\beta} - \alpha\sqrt{\bar{e}^i}\sqrt{\bar{e}^j}K^{i\beta}K^{j\beta} - p^e\bar{e}^i - \frac{1}{2}cI^{i2}. \tag{15}$$

The derivative of marginal profits to foreign capital, denoted by Π^i_{ij}, is negative for both policy instruments and for all values of capital and energy use. The derivative of marginal profits to own capital, Π^i_{ii}, is assumed to be negative. For standards, this requires p_0 to be large enough, for taxes it requires an upper limit on $(p^e+\tau)$. It can be derived that the second-order derivative of profits to foreign capital, Π^i_{jj}, is always positive under standards. Under taxes, $\Pi^i_{jj} > 0$ requires again that $(p^e+\tau)$ is not too large. In the sequel it is assumed that this is the case.
The objective functions are approximated with revenue functions that are linear-quadratic in (K^i, K^j). For these objective functions, we suppose that this gives a reasonable approximation to the feedback steady-state solution. The approximation is found by application of the following algorithm:

step 1) Choose a starting point, (K_0, K_0).

step 2) Compute a second-order Taylor approximation of the objective function in the neighbourhood of this starting point.

step 3) Determine the steady-state capital stocks for this approximation analytically.

step 4) Take the resulting steady state as the new starting point and return to step 2. Repeat the algorithm until the new steady state is close enough to the old one.

When convergence occurs, this is at the point (K^*, K^*) which is the steady-state capital stock for the Taylor approximation of the objective function around that same point, (K^*, K^*). In appendix A, step 2 and 3 of this algorithm are elaborated.
With the above algorithm, we can compute linear equilibrium investment strategies, $I(K^i, K^j) = P_1K^i + P_3K^j + P_4$, and steady-state capital stocks, $\hat{K}$, under taxes and s-tandards, for a given target of environmental policy, $\bar{e}$. These strategies are approximations to the feedback equilibrium strategies in a neighbourhood of (K^*, K^*).
The sign of P_3 is interesting, because it determines the type of strategic interaction in case of feedback investment strategies. If $P_3 > 0$ a firm reacts to a higher capital stock of its competitor with higher investments. If $P_3 < 0$ the reverse is the case. The following proposition shows that the only possibly stable solution has a negative P_3 under a certain condition on the second-order derivatives of (14), respectively (15). Furthermore a sufficient condition for this solution to be stable is given. These conditions can be shown to hold for reasonable values of the parameters both for taxes and for standards. The proof of the proposition is given in appendix B.

Proposition 3.1 *For*

$$\Pi^i_{jj} \leq \frac{1}{4}(2\delta + r)^2 c + 2\Pi^i_{ij} - \Pi^i_{ii} \tag{16}$$

one or more solutions exist. Only one solution can be stable and for this solution $P_3 < 0$. If, in addition,

$$\Pi^i_{ij} < min[-\frac{1}{2}r^2 c, -\frac{1}{4}\sqrt{6}], \tag{17}$$

then this solution is indeed stable.

In other words, under the conditions of the proposition, a unique stable feedback equilibrium steady-state capital stock, (K^*, K^*), exists for the approximated objective function and the corresponding equilibrium investment strategy has $P_3 < 0$. Under taxes, both conditions in the proposition are satisfied, for all β, given the other parameter values used. Under standards, the conditions in the proposition are satisfied for most parameter values. However, it may happen that $\Pi^i_{jj} > \frac{1}{4}(2\delta + r)^2 c + 2\Pi^i_{ij} - \Pi^i_{ii}$. In that case, multiple stable equilibria may exist and there is a coordination problem on the choice of equilibrium strategies. For β and $\bar{e}$ low enough, however, this case can be excluded (see appendix C). Therefore, for both taxes and standards the effect of foreign capital on own investments, $\frac{\partial I^{iFB}}{\partial K^j} = P_3$, is negative around the steady state, if the parameter values are not extreme.

Given the decreasing marginal productivity of capital it is to be expected that the derivative of investment with respect to own capital, $\frac{\partial I^{iFB}}{\partial K^i} = P_1$, is negative. This is indeed true for realistic values of δ, r and c, which is formally derived in appendix D. For -unreasonably- large values of the parameters mentioned, it is possible to find a positive P_1, due to indirect effects of capital on equilibrium emissions.

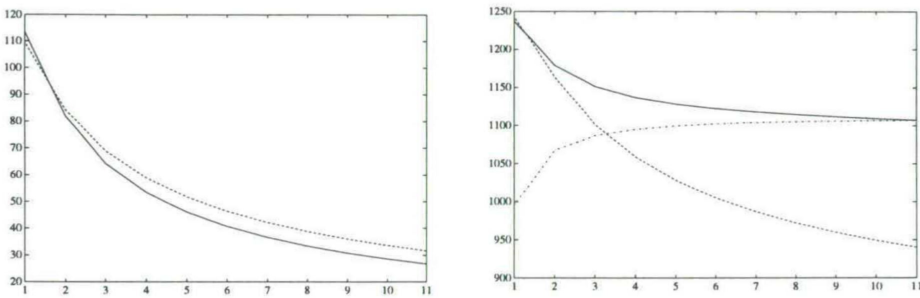
Taxes and standards are to be compared in the feedback equilibrium steady state. Figure 1 shows steady-state capital stocks and firm profits for both policy instruments as a function of the environmental target. Before-tax profits are also shown, to enable a good comparison of taxes and standards. Of course, after-tax profits are always substantially lowered by the tax-payment. From a welfare perspective, it is better to compare before-tax profits with profits under standards.

Remember that in this model, due to rivalry between firms, investment is too high, so that the higher the capital stock, the lower profits. Looking at the example, it is clearly not true that taxes always result in higher capital stocks than standards. On the contrary, for most parameter values, standards result in higher capital stocks. That is due to the investments that are carried out for strategic reasons. As a consequence profits are larger under taxes for most parameter values, even when after-tax profits are considered. The value of β is an important parameter in this respect, as well as the stringency of environmental policy. The laxer environmental policy, the more likely it is that standards result in higher steady-state capital stocks and lower profits than taxes[3].

[3]Provided that a unique stable equilibrium exists.

Figure 1: Capital stocks and profits as a function of the environmental target. (Parametervalues: $\beta = 0.7$, $c = 2$, $\delta = 0.10$, $r = 0.08$, $p^e = 1$, $\alpha = 1$. Notation: - - - = standards, —— = taxes/before-tax profits, - . - = after-tax profits.)

Capital stocks: Profits:



$K \uparrow e \rightarrow$ $\Pi \uparrow e \rightarrow$

3.3 Strategic effects

To explain the effect of strategic behaviour on investment under environmental policy, we use the so called net present value expressions. These expressions can be derived from forward integration of first-order conditions. They give the properly discounted future stream of extra profits due to an additional unit of capital at time t (See for instance Hartl and Kort, 1996).
First, consider the cooperative solution. From the point of view of the two firms as a cartel, this is the optimal outcome. It is found as the solution to the following optimal-control model:

$$\max_{I^i \geq 0, I^j \geq 0} \int_0^\infty e^{-rt}[\Pi^i + \Pi^j]dt \tag{18}$$

$$\text{s.t. } \dot{K}^i = I^i - \delta K^i, \quad \text{for } i = 1, 2 \tag{19}$$

with $e^i = e^{iC}$. From the first-order conditions a net present value expression can be derived, which says that optimal investment is characterized by:

$$\int_t^\infty e^{-(r+\delta)(s-t)}[\frac{\partial R^i}{\partial x^i}\frac{\partial x^i}{\partial K^i} + \frac{\partial R^j}{\partial x^i}\frac{\partial x^i}{\partial K^i}]ds - C'(I^i(t)) = 0 \tag{20}$$

The term in brackets gives the extra revenues at time s due to an additional unit of capital stock, invested at time t. Future revenues are discounted with rate r and corrected for depreciation at rate δ, since one unit of capital bought at time t reduces in value to $e^{(r-\delta)(s-t)}$ units at time s. To obtain this unit, the firm must spend $C'(I^i)$ at time t. In case of cooperation, the -negative- effect of increased home capital on earnings of the foreign firm is taken into account. All

strategic interaction is absent, since the firms cooperate.
Second, consider equilibria in which the firms compete in a Cournot-Nash fashion. The net present value expression for firms that choose investments according to an open-loop strategy was derived in chapter 2:

$$\int_t^\infty e^{-(r+\delta)(s-t)}[\frac{\partial R^i}{\partial x^i}\frac{\partial x^i}{\partial K^i} + \frac{\partial R^i}{\partial x^j}\frac{\partial x^j}{\partial e^j}\frac{\partial e^{jN}}{\partial K^i}]ds - C'(I^i(t)) = 0 \tag{21}$$

This expression contains the term, $\frac{\partial R^i}{\partial x^j}\frac{\partial x^j}{\partial e^j}\frac{\partial e^{jN}}{\partial K^i}$, due to strategic interaction. Firms try to influence energy and, hence, output decisions of their competitor with their capital stock. If they manage to decrease foreign output, their own revenues increase ($\frac{\partial R^i}{\partial x^j} < 0$). Since $\frac{\partial x^j}{\partial e^j}\frac{\partial e^{jN}}{\partial K^i} < 0$, when taxes are applied as an instrument of environmental policy, this strategic effect leads to more investment. In case of environmental standards, however, $\frac{\partial e^{jN}}{\partial K^i} = 0$ because firms are on the boundary of a binding constraint. In that case the strategic effect is absent and investments are lower.
Finally consider the net present value expression when firms use feedback investment strategies. This requires first-order conditions for such an equilibrium, which can for instance be found in Feichtinger and Hartl (1986, p.536). In our model this leads to:

$$C'(I^i) = \mu^{ii} \tag{22}$$

$$\dot{\mu}^{ii} = (r+d)\mu^{ii} - [\frac{\partial R^i}{\partial x^i}\frac{\partial x^i}{\partial K^i} + \frac{\partial R^i}{\partial x^j}\frac{\partial x^j}{\partial e^j}\frac{\partial e^{jN}}{\partial K^i} + \mu^{ij}\frac{\partial I^j}{\partial K^i}] \tag{23}$$

$$\dot{\mu}^{ij} = (r+d)\mu^{ij} - \mu^{ij}\frac{\partial I^j}{\partial K^j} - [\frac{\partial R^i}{\partial x^j}\frac{\partial x^j}{\partial K^j} + \frac{\partial R^i}{\partial x^j}\frac{\partial x^j}{\partial e^j}\frac{\partial e^{jN}}{\partial K^j}] \tag{24}$$

Here μ^{ii} denotes the shadow value of own capital to firm i. Its value is determined by condition (23). This includes the term $\mu^{ij}\frac{\partial I^j}{\partial K^i}$ because of feedback reactions. The variable μ^{ij} denotes the shadow value of foreign capital to firm i. Condition (24) determines its value. Note that these first-order conditions are not sufficient to actually compute candidate solutions, since in general $\frac{\partial I^i}{\partial K^j}$ and $\frac{\partial I^j}{\partial K^j}$ are not known. Integrate forward and rearrange, to find the following net present value expression:

$$\int_t^\infty e^{-(r+\delta)(s-t)}[\frac{\partial R^i}{\partial x^i}\frac{\partial x^i}{\partial K^i} + \frac{\partial R^i}{\partial x^j}\frac{\partial x^j}{\partial e^j}\frac{\partial e^{jN}}{\partial K^i} + \mu^{ij}\frac{\partial I^j}{\partial K^i}]ds - C'(I^i(t)) = 0 \tag{25}$$

with μ^{ij} given by

$$\mu^{ij}(s) = \int_s^\infty e^{-(r+\delta-\frac{\partial I^j}{\partial K^j})(u-s)}[\frac{\partial R^i}{\partial x^j}\frac{\partial x^j}{\partial K^j} + \frac{\partial R^i}{\partial x^j}\frac{\partial x^j}{\partial e^j}\frac{\partial e^{jN}}{\partial K^j}]du \tag{26}$$

In case of standards in both countries, both firms are at a corner solution for energy use. Therefore, these formulas then become:

$$\int_t^\infty e^{-(r+\delta)(s-t)}[\frac{\partial R^i}{\partial x^i}\frac{\partial x^i}{\partial K^i} + \mu^{ij}\frac{\partial I^j}{\partial K^i}]ds - C'(I^i(t)) = 0 \tag{27}$$

with μ^{ij} given by

$$\mu^{ij}(s) = \int_s^\infty e^{-(r+\delta-\frac{\partial I^j}{\partial K^j})(u-s)}[\frac{\partial R^i}{\partial x^j}\frac{\partial x^j}{\partial K^j}]du \tag{28}$$

Compared to (21), an additional strategic effect exists that works through investment, $\mu^{ij}\frac{\partial I^j}{\partial K^i}$. This effect can be strong enough to outweigh the first strategic effect. From a comparison of the net present value expressions follows immediately:

Proposition 3.2 *When* $\frac{\partial R^i}{\partial x^j}\frac{\partial x^j}{\partial e^j}\frac{\partial e^{j\,N}}{\partial K^i} + \mu^{ij}(tax)\frac{\partial I^j}{\partial K^i} < \mu^{ij}(sta)\frac{\partial I^j}{\partial K^i}$ *for all t, the strategic incentive for investment is greater under standards than under taxes.*

The term $\mu^{ij}\frac{\partial I^j}{\partial K^i}$ captures the indirect strategic effect. If firm i increases its investment, its capital stock grows. This influences the investment by firm j immediately as expressed by $\frac{\partial I^j}{\partial K^i}$. The shadow price μ^{ij} denotes the valuation by firm i of such a change in firm j's investment. Thus μ^{ij} gives the valuation by firm i of an additional amount of capital owned by firm j. This is given by the properly discounted flow of marginal decreases in firm i's revenues if firm j owns an extra amount of capital (cf. (26) and (28)). Since firm i's revenues decrease when competition from firm j increases, μ^{ij} will have a negative sign. If the derivative of I^j to K^i is negative, which we have shown to be true for reasonable parameter values (cf. proposition 3.1), then the additional strategic effect is positive. This effect, $\mu^{ij}\frac{\partial I^j}{\partial K^i}$, can be higher for standards than for taxes, since firms are less flexible in case of binding standards and do not substitute energy for capital. To explain this, first note that with feedback investment strategies, given that $\frac{\partial I^j}{\partial K^i} < 0$, a marginal increase in capital K^i leads firm j to invest less and hence to decrease its capital stock, K^j. This in turn leads firm j to decrease its output. It depends on the sign of $\frac{\partial e^{j\,N}}{\partial K^j}$ whether j's use of energy also decreases. If substitution effects dominate, e^j increases. In case of standards, substitution between production factors - that is, an increase in e^j in reaction to a decrease in K^j - will not occur, since the standard is an upper bound to e^j. A decrease in e^j will not happen since the firm is at a corner solution for its energy use. Hence, $\frac{\partial e^{j\,N}}{\partial K^j}$ equals zero. In case of taxes, on the contrary, the firm is flexible to adjust its use of energy in an optimal way to the capital stocks. Substitution between energy and capital then partly cancels the effect on output of an increased capital stock. That implies that the change in firm j's output and hence the relevant strategic effect, $\mu^{ij}\frac{\partial I^j}{\partial K^i}$, can be greater with standards than with taxes, provided substitution effects dominate marginal-productivity effects.

The proposition contradicts earlier conclusions by Ulph (1992). He derived that standards always result in less strategic investments than taxes. Ulph used a multistage model. That implied that his subgame-perfect equilibrium is equivalent to a Nash equilibrium with feedback strategies for energy use, but open-loop strategies for investment, as represented by the net present value expression (21). His model does not allow for strategic interaction between firms in investment strategies. Only then the result holds that taxes always provide larger incentives for investment than standards.

To conclude, it is not generally true that standards moderate strategic overinvestment when firms react on each others behaviour by adjustments in their investment plans. Although standards commit firms to a certain use of energy input, they may drive firms to more investment than taxes. In that case the use of standards as a commitment device does not work. Then, the use of taxes as an environmental policy instrument is to be preferred when, next to environmental targets, profits of domestic firms are an important objective to the government.

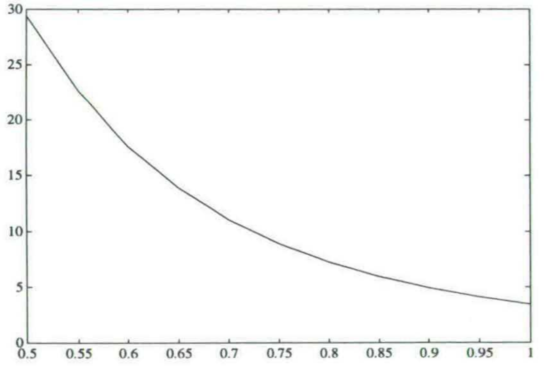
4 Comparative statics for parameter values

Figure 1 shows that standards may result in more strategic investment than taxes. As a consequence, firm profits are higher under taxes than under standards. For some strict environmental policy targets the reverse is true, however. This section discusses the values chosen for the parameters that may influence this result. We show some comparative static results that indicate how the difference between taxes and standards changes with parameter values. The relationship between the parameters of the model and the steady-state values of the capital stock in the feedback solution is complex. It involves the approximation algorithm and the computation of the roots of a third degree polynomial in the third step of this approximation algorithm. Therefore we cannot give simple expressions that link parameter values to the conditions in the propositions. But numerical experimentation gives some clues about the direction of changes.
First consider the parameter β in the production function. The literature on econometric production functions provides empirical estimates of returns to scale in large energy-intensive industries. These gave a range around 1.2 (Morrison, 1993, 1994, Ilmakunnas and Törmä, 1994, Pindyck and Rotemberg, 1983). To satisfy the model assumptions, $\frac{1}{2} < \beta < 1$ must hold. Given that the Cobb-Douglas coefficient of energy equals $\frac{1}{2}$, returns to scale then vary between 1 and 1.5. Compared to 1.2, this seems reasonable given the empirical literature.
Now consider the price of energy, p^e. When prices are extremely high, they provide enough incentives for firms to reduce energy use and environmental policy is superfluous. Therefore, we took the relatively low value of 1 for this price. In reality, of course, energy prices are highly volatile, but this is only a numerical example, meant to provide insight into the equilibrium solution in case of feedback investment strategies.

Figure 2 depicts emissions as a function of β, when no environmental policy is applied and p^e has the value 1. It shows that for values of β close to 1, even with this low value for p^e, the range of meaningful[4] emission goals is limited. When β is high, energy is a relatively unimportant production factor, and for lax environmental policy goals, environmental policy is unnecessary. The higher the price of energy, p^e, the sooner (in terms of increasing β) this will happen.
For δ, the rate of depreciation, we took the value 0.10. For r, the rate of discount we took the value 0.08. Figure 3 shows the effect of different values for δ. If δ is higher, capital depreciates faster. That implies that commitment with regard to future output provided by an

[4]In the sense that positive taxes and binding emission standards are required to enforce these goals.

Figure 2: Emissions for no policy case, as a function of β. (Parametervalues: $c = 2$, $\delta = 0.10$, $r = 0.08$, $p^e = 1$, $\alpha = 1$.)



$e \uparrow \quad \beta \rightarrow$

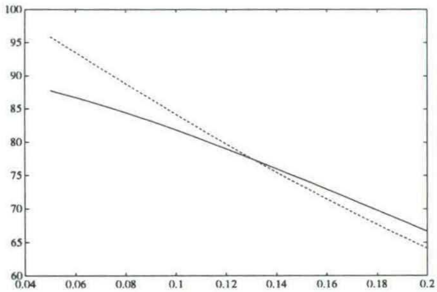
additional unit of capital decreases. The consequence is a lower steady-state capital stock and a smaller difference between standards and taxes. For δ high enough, the difference between standards and taxes changes sign, because strategic interaction through commitment on energy use dominates strategic interaction through commitment on investments. Then taxes result in more strategic investment than standards. Increases in r have a similar effect. A higher rate of discount implies that the future is less important relative to the present. Current costs of investment then become important relative to future earnings. The steady-state capital stock hence decreases. The dynamic aspects that cause a difference between environmental policy instruments in strategic interaction through commitment on investment loose importance. As a result, the difference between standards and taxes decreases and eventually changes sign, when the static aspect of strategic interaction through commitment on energy use starts to dominate.

For c, that is a parameter for adjustment costs of investment, we took the value 2. The higher c, the higher the costs of investment, and the lower therefore the steady-state stock of capital. Changes in capital stock become more expensive, so that commitment through investment is less attractive. Therefore the differences between standards and taxes decrease with c.

The parameter α, finally, denotes the interconnectedness of markets. If $\alpha = 0$, firms do not influence each others prices. In that case both firms are monopolists and strategic effects disappear. Environmental taxes and standards then have equal effects on firms in the model above. The higher α, the more effect firms have on each others prices and the more important strategic effects become. For $\alpha = 1$ the outputs of the two firms are perfect substitutes. Figure 4 shows equilibrium capital stocks and profit rates for different values of α. The difference between standards and taxes increases with α, because strategic effects gain importance. We took α to be equal to 1 in the other figures.

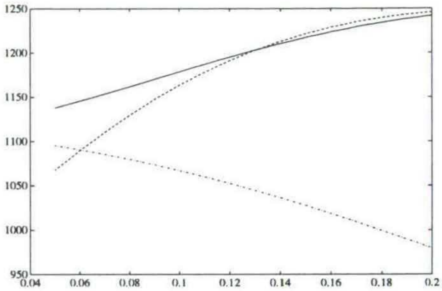
Figure 3: Effect on capital stock and profits of changes in δ. (Parametervalues: $\beta = 0.7$, $c = 2$, $r = 0.08$, $p^e = 1$, $\alpha = 1$. Notation: - - - = standards, — = taxes/before-tax profits, - . - = after-tax profits.)

Capital stocks:



K↑ $\delta \rightarrow$

Profits:



Π↑ $\delta \rightarrow$

The results in this section show again that it is not true that taxes always result in more investment than standards for strategic reasons, and therefore in lower profits. The reverse, that standards lead to more investment and lower profits is neither valid. Which environmental policy instrument is 'better', which means here that it results in the highest profits for a given emission target, depends on the values of the parameters.

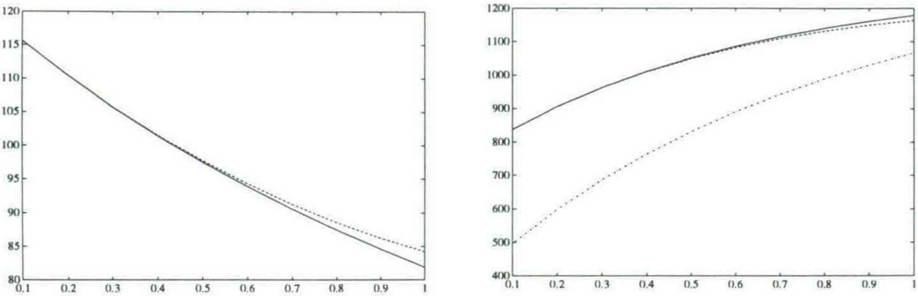
5 Conclusions

This chapter is concerned with international rivalry and environmental policy. In a multistage framework Ulph (1992) found that environmental taxes lead to higher investment than standards. Firms that compete on an international market have an incentive to increase investments to gain strategic advantage. But in equilibrium, competitors act similar so that higher capital stocks, more output and lower profits result. Therefore, ceteris paribus, governments prefer standards rather than taxes, due to the reduction in strategic investment. This was confirmed in the previous chapter in a differential-game framework with open-loop investment strategies. The differences between environmental taxes and standards as regards their effect on strategic behaviour are due to their influence on the flexibility of firms. Standards reduce a firm's flexibility in its choice of emissions, while taxes do not.

In this chapter, flexibility in investment behaviour is introduced. More specifically, we consider feedback investment strategies. Under feedback investment strategies, firms have to take into account that the competitor will react through its investment on marginal increases in capital stocks. The effects of environmental standards and taxes on this type of strategic behaviour

Figure 4: Effect on capital stock and profits of changes in α. (Parametervalues: $\beta = 0.7$, $c = 2$, $\delta = 0.10$, $r = 0.08$, $p^e = 1$. Notation: - - - = standards, —— = taxes/before-tax profits, - . - = after-tax profits.)

Capital stocks: Profits:



K$\uparrow$ $\alpha \rightarrow$ $\Pi \uparrow \alpha \rightarrow$

differ. It turns out that it cannot -as in the case of open-loop investment strategies- be stated that taxes will always lead to more investments than standards. It is shown in the chapter that if substitution effects between production factors are large enough, investment is larger under standards than under taxes. Ceteris paribus, governments then prefer taxes rather than standards as environmental policy instrument.

Note that in the current model, consumer surpluses are neglected. It is implicitly assumed that firms sell a substantial part of their output to third countries. Ulph (1996a) and Kennedy (1994) include consumer surplus in a multistage game. Since it is in the interest of consumers that competition between the two firms results in more output, they favour larger investments. Inclusion of consumer surplus would therefore require a different valuation of high capital stocks. When consumer surplus is important, taxes may be preferred by the government to standards, even while taxes lead to more investment than standards.

A differential game model of a duopoly describes the behaviour of firms that compete in an international market. After equilibrium behaviour on energy choice has been inserted, a capital accumulation game results. The two firms choose investment rates, to maximize their objectives, given government policy and the strategy of the competitor. Feedback equilibria in capital accumulation games are derived in Reynolds (1987) and in Fershtman and de Zeeuw (1992). The game in this paper is a bit different, because investment has indirect effects on output through energy use. That implies that the objective function is not linear quadratic in capital stocks like in the two papers mentioned above. But it is possible with the help of an approximation algorithm to extend the method employed in Fershtman and de Zeeuw and to find approximations to the steady state.

The indirect effects through energy choices and the approximation step imply that equilibrium

investment depends in a complex way on domestic and foreign capital stocks. In particular, the sign of the derivative of equilibrium investment to foreign capital may be either negative or positive. Conditions are given for stable equilibria that are characterized by a negative sign of this derivative. Although these conditions capture the cases that are economically most relevant, exceptional cases with positive derivatives are still possible.
Flaherty (1980) analyses a capital accumulation game with symmetric and asymmetric equilibria. With the help of linear approximations, it is shown that the asymmetric equilibria are stable and the symmetric equilibrium is not. Although in a different context, this shows that stable asymmetric equilibria are certainly an option in this type of models. For tractability reasons, our analysis has been restricted to symmetric equilibria for a symmetric model, in which both countries apply the same type of environmental policy. Extensions to the asymmetric case where one country applies taxes and the other standards is an option. However, we do not expect that a (complex) analysis of the asymmetric case would change our conclusion with respect to strategic behaviour.

A Details of the algorithm

In this appendix details are given about the 2^{nd} and 3^{rd} step of the algorithm in the main text.

Step 2

A second order Taylor approximation around a point (K_0, K_0) of the objective function of the differential game results in a profit function:

$$\Pi^i = \frac{1}{2}K^tQK + q^tK + q_0 - \frac{1}{2}cI_i^2 \tag{A.1}$$

where Q is a symmetric matrix $[\begin{matrix} Q_1 & Q_3 \\ Q_3 & Q_2 \end{matrix}]$, q is a column vector (q_1, q_2), q_0 is a scalar and K is a column vector (K^1, K^2). The Q_i denote the second derivatives of profits to capital, respectively $Q_1 = \Pi^i_{ii}$, $Q_2 = \Pi^i_{jj}$ and $Q_3 = \Pi^i_{ij}$. Furthermore,

$$q_1 = \Pi^i_i - K_0Q_1 - K_0Q_3$$

$$q_2 = \Pi^i_j - K_0Q_3 - K_0Q_2.$$

Step 3

For a linear-quadratic objective function, the analytical expression of the steady state can be computed. A model with $Q_2 = 0$, $Q_1 = -2$, $Q_3 = -1$, $q_2 = 0$ and $q_1 = a > 0$, has been solved by Reynolds

(1987) and by Fershtman and de Zeeuw (1992). Below, the approach of the latter paper is followed to obtain a solution to the approximated game.

Consider the differential game with Π^i given by (A.1) as the objective function. This is a game with a quadratic objective function, linear dynamics and two state variables. The dynamic-programming approach is used to find a linear feedback Nash equilibrium. The Hamilton-Jacobi-Bellman equations for the game are:

$$rV^i(K^i, K^j) =$$

$$\max_{I^i}[-\frac{1}{2}cI^{i^2} + \frac{1}{2}K^tQK + q^tK + q_0 + V^i_{K^i}(I^i - \delta K^i) + V^i_{K^j}(I^j - \delta K^j)] \tag{A.2}$$

The optimal I^i is given by $I^i = \frac{1}{c}V^i_{K^i}$. A quadratic value function has the form:

$$V^i = \frac{1}{2}K^tPK + p^tK + p_0 \tag{A.3}$$

with P a symmetric matrix $\begin{bmatrix} P_1 & P_3 \\ P_3 & P_2 \end{bmatrix}$, p a column-vector (p_4, p_5) and p_0 a scalar. The partial derivatives of the value function become:

$$V^i_{K^i} = P_1K^i + P_3K^j + p_4 \tag{A.4}$$

and

$$V^i_{K^j} = P_3K^i + P_2K^j + p_5 \tag{A.5}$$

Hence the optimal investment strategy can be written:

$$I^i = \frac{1}{c}(P_1K^i + P_3K^j + p_4) \tag{A.6}$$

Because of symmetry, it suffices to consider the Hamilton-Jacobi-Bellman equation of only one firm. For the other firm the computations are analogous. Insert the optimal solution for investment, given by (A.6), and the value function (A.3), with derivatives (A.4) and (A.5), into equation (A.2) and note that it must hold for each K. Equations for the elements of P and p result if the coefficients of the quadratic expression in K are equated to zero:

$$P_1^2 + 2P_3^2 - (2\delta + r)cP_1 + Q_1c = 0 \tag{A.7}$$

$$P_3^2 + 2P_1P_2 - (2\delta + r)cP_2 + Q_2c = 0 \tag{A.8}$$

$$2P_3P_1 + P_3P_2 - (2\delta + r)cP_3 + Q_3c = 0 \tag{A.9}$$

$$P_1p_4 + P_3p_4 + P_3p_5 - c(\delta + r)p_4 + q_1c = 0 \tag{A.10}$$

$$P_3p_4 + P_2p_4 + P_1p_5 - c(\delta + r)p_5 + q_2c = 0 \tag{A.11}$$

$$p_4^2 + 2p_4p_5 - 2cq_0 - 2crp_0 = 0 \tag{A.12}$$

If the investments strategies (A.6) must also result in a stable capital growth path, two stability conditions follow from solving for (13) for i=1,2 with (A.6):

$$P_1 + P_3 - \delta c < 0 \tag{A.13}$$

$$P_1 - P_3 - \delta c < 0 \tag{A.14}$$

The first three equations, (A.7) to (A.9), form an independent system of equations in P_1, P_2 and P_3. These equations define conic sections in the (P_1, P_3)-plane, respectively an ellipse, a parabola and a hyperbola. This suggests to apply polar coordinates. First note that the ellipse has its centre at $(\frac{1}{2}(2\delta + r)c, 0)$, the parabola its top at $(\frac{1}{2}(2\delta + r)c - \frac{Q_2c}{2P_2}, 0)$ and the hyperbola its centre at $(\frac{1}{2}(2\delta + r)c - \frac{1}{2}P_2, 0)$. With the transformation $p_1 = P_1 - \frac{1}{2}(2\delta + r)c$ and $p_3 = \sqrt{2}P_3$ the three equations simplify to:

$$\frac{p_1^2}{\rho^2} + \frac{p_3^2}{\rho^2} = 1 \tag{A.15}$$

$$\frac{1}{2}p_3^2 + 2P_2p_1 + Q_2c = 0 \tag{A.16}$$

$$\sqrt{2}p_3p_1 + \frac{1}{2}\sqrt{2}p_3P_2 + Q_3c = 0 \tag{A.17}$$

with

$$\rho^2 = \frac{1}{4}(2\delta + r)^2c^2 - Q_1c \tag{A.18}$$

Equation (A.15) defines a circle with radius ρ and centre at the origin. The polar coordinates R and ϕ are introduced in the (p_1, p_3) plane (with origin $(\frac{1}{2}(2\delta + r)c, 0)$ in the (P_1, P_3) plane):

$$p_1 = R\sin\phi \tag{A.19}$$

$$p_3 = R\cos\phi \tag{A.20}$$

so that in the original variables:

$$P_1 = \frac{1}{2}(2\delta + r)c + R\sin\phi \tag{A.21}$$

$$P_3 = \frac{1}{2}\sqrt{2}R\cos\phi \tag{A.22}$$

with $-\Pi < \phi < \Pi$. The P_2 axis is left unchanged.
From equation (A.15) it follows that

$$R^2 = \rho^2. \tag{A.23}$$

Rewrite equation (A.16) to an expression for P_2, do the same with equation (A.17) and equate these two:

$$\frac{-(2Q_2c + p_3^2)}{4p_1} = \frac{-(\sqrt{2}Q_3c + 2p_1p_3)}{p_3}. \tag{A.24}$$

Now insert (A.23), (A.19) and (A.20) in (A.24). Divide by $\cos\phi$ and realize that $\frac{\sin\phi}{\cos\phi} = \tan\phi$ and that $\cos^2\phi = \frac{1}{\tan^2\phi+1}$. After multiplication with $(1+\tan^2\phi)$ and division by $-4\sqrt{2}Q_3c$ the following equation results:

$$\tan^3\phi + (\frac{-8\rho^2 + 2Q_2c}{-4\sqrt{2}Q_3c})\tan^2\phi + \tan\phi + (\frac{2Q_2c + \rho^2}{-4\sqrt{2}Q_3c}) = 0 \tag{A.25}$$

With the notation $y = \tan\phi$, $u = \frac{-\sqrt{2}\rho^2}{Q_3c}$, $v = \frac{-Q_2}{Q_3}\frac{1}{4}\sqrt{2}$, this can be rewritten as:

$$y^3 + (-u+v)y^2 + y + (\frac{1}{8}u + v) = 0 \tag{A.26}$$

This is a third degree polynomial. Its roots, y_1, y_2 and y_3 may be real or complex, dependent on the values of u and v. The three roots are given by the formulas:

$$y_1(u,v) = \frac{1}{3}(u-v) + 2\sqrt{-q}\cos(\frac{\psi}{3}) \tag{A.27}$$

$$y_2(u,v) = \frac{1}{3}(u-v) + 2\sqrt{-q}\cos(\frac{\psi}{3} + \frac{2\Pi}{3}) \tag{A.28}$$

$$y_3(u,v) = \frac{1}{3}(u-v) + 2\sqrt{-q}\cos(\frac{\psi}{3} - \frac{2\Pi}{3}) \tag{A.29}$$

$$q = \frac{1}{3} - \frac{1}{9}u^2 + \frac{1}{9}(2uv - v^2) \tag{A.30}$$

$$r = \frac{u}{3}(\frac{u^2}{9} - \frac{11}{16}) + \frac{v}{3}(\frac{uv}{3} - \frac{u^2}{3} - \frac{v^2}{9} - 1) \tag{A.31}$$

where ψ is defined by

$$\cos\psi = \frac{r}{(-q)^{\frac{3}{2}}} \tag{A.32}$$

and

$$0 < \psi \leq \Pi \tag{A.33}$$

Stable solutions for the feedback equilibrium are obtained from roots that after the appropriate transformations[5] result in P_1, P_2 and P_3 that satisfy the stability requirements (A.13) and (A.14). In polar coordinates these stability conditions read:

$$\rho[\sin\phi + \frac{1}{2}\sqrt{2}\cos\phi] < -\frac{1}{2}rc \tag{A.34}$$

and

$$\rho[\sin\phi - \frac{1}{2}\sqrt{2}\cos\phi] < -\frac{1}{2}rc. \tag{A.35}$$

These can only be satisfied simultaneously by $-\Pi < \phi < 0$. Therefore, the interval $0 \leq \phi < \Pi$ can be excluded. The stability conditions (A.34) and (A.35) can be rewritten to:

$$(1 - \frac{r^2c^2}{4\rho^2})\tan^2\phi + \sqrt{(2)}\tan\phi + \frac{1}{2} - \frac{r^2c^2}{4\rho^2} > 0 \tag{A.36}$$

$$(1 - \frac{r^2c^2}{4\rho^2})\tan^2\phi - \sqrt{(2)}\tan\phi + \frac{1}{2} - \frac{r^2c^2}{4\rho^2} > 0 \tag{A.37}$$

Note that $4\rho^2 - r^2c^2 > 0$ Let b_1, b_2, respectively b_3, b_4 denote the zeros of the two polynomials in $\tan\phi$. It can be proved that $b_2 > -\frac{1}{2}\sqrt{2}$ and $b_3 < \frac{1}{2}\sqrt{2}$. Also, if $|\tan\phi| < \frac{1}{2}\sqrt{2}$ then either $0 > \sin\phi \geq -\frac{1}{2}\sqrt{2}\cos\phi$ and (A.34) cannot be satisfied or $\frac{1}{2}\sqrt{2}\cos\phi \leq \sin\phi < 0$ and (A.35) cannot be satisfied. It follows that

$$\tan\phi < b_1 \text{ or } b_4 < \tan\phi \tag{A.38}$$

is a necessary condition for stability. Only $y_i = \tan\phi$ that satisfy condition (A.38) result in a stable solution.

If there is only one such a real root, a corresponding unique feedback equilibrium results. If more roots satisfy the stability requirements a situation of multiple equilibria occurs. Whether a unique root that satisfies (A.38) exists, depends on the values of Q_1, Q_2 and Q_3 in the objective function.

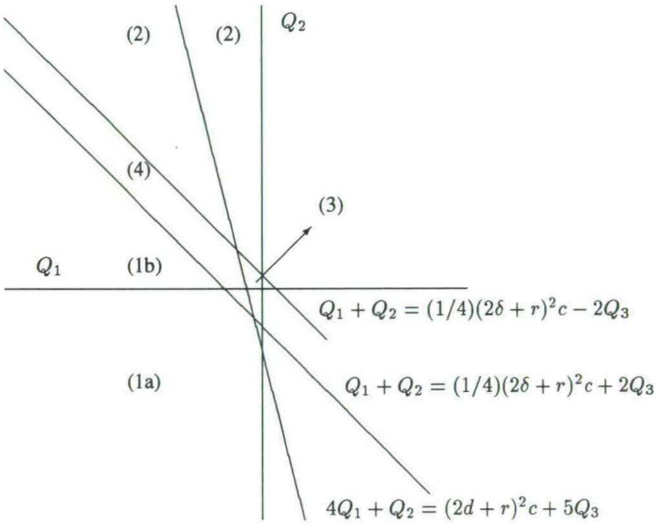
In figure 5 the (Q_1, Q_2) plane is divided in 4 different regions defined by:

(1) $Q_1 + Q_2 < \frac{1}{4}(2\delta + r)^2c + 2Q_3$

(2) $Q_1 + Q_2 > \frac{1}{4}(2\delta + r)^2c - 2Q_3$

[5]take $\phi = \arctan y_i$ and compute P_1, P_2 and P_3 with the help of (A.21) and (A.22)

Figure 5: Division of Q_1, Q_2-plane, for given Q_3.



(3) $\frac{1}{4}(2\delta + r)^2 c + 2Q_3 < Q_1 + Q_2 < \frac{1}{4}(2\delta + r)^2 c - 2Q_3$ and

$4Q_1 + Q_2 < (2\delta + r)^2 c + 5Q_3$

(4) $\frac{1}{4}(2\delta + r)^2 c + 2Q_3 < Q_1 + Q_2 < \frac{1}{4}(2\delta + r)^2 c - 2Q_3$ and

$4Q_1 + Q_2 > (2\delta + r)^2 c + 5Q_3$

In region (1), one real root of (A.26) that is larger than $\frac{1}{2}\sqrt{2}$ exists. Therefore, in this region there can be at most one stable solution. There is none if $y_1 < b_4$. In region (2), one real root with $y_i < -\frac{1}{2}\sqrt{2}$ exists, while other roots are not stable. Hence in this region at most one stable root exists, but none if $y_2 > b1$. In region (3) all roots satisfy $b1 \leq |y_i| \leq b_4$ and therefore no stable solution exists. In region (4), two roots with $y_i > \frac{1}{2}\sqrt{2}$ exist. When these are both larger than b_4 multiple stable equilibria exist. For any unique root $\hat{y}$ that satisfies the stability criteria, we find $\hat{\phi} = \arctan(\hat{y})$, with $-\Pi < \hat{\phi} < 0$. Together with the value for ρ from equation (A.23), this results in values for P_1 and P_3. Equation (A.8), or alternatively (A.9), gives P_2. Finally equations (A.10) to (A.12) determine p_4, p_5 and p_0. From (A.6) then follow equilibrium linear feedback investment strategies. The steady-state value of capital is found as the solution to the system of equations (13) in the steady state:

$$\bar{K}^{iFB} = \frac{-p4}{P1 + P3 - \delta c}. \tag{A.39}$$

The equilibrium path of capital becomes:

$$K(t) = (K_0 - \bar{K}^{FB})e^{\frac{P_1+P_3-\delta c}{c}t} + \bar{K}^{FB}. \tag{A.40}$$

We have found an analytic solution to the problem for a Taylor approximation of the original objective function around the point (K_0, K_0). The equilibrium steady-state will be the new starting point (K_0, K_0) of the algorithm. This procedure is repeated until the resulting steady state is close enough to the starting point.

B Proof of proposition 1

See figure 5 above and remember that Q_1, Q_2 and Q_3 denote Π^i_{ii}, Π^i_{jj} and Π^i_{ij} in the point (K^*, K^*). Also note that if the root of equation (A.26), y_i, is positive (negative) then, from (A.22), $P_3 = \frac{1}{2}\rho\sqrt{2}\ \cos[\arctan(y_i)]$ is negative (positive).

In appendix A it is shown that for region (1) in figure 5 at most one stable root of (A.26) exists. This region can be split up in a part with $Q_2 < 0$ (region 1a) and one with $Q_2 \geq 0$ (region 1b). From the assumptions made below equation (15), it follows that only the latter region is relevant for proposition 1. It will be shown that indeed in region (1b) proposition 1 holds. That is, under the conditions in the proposition $y_1 > b_4$, so that y_1 satisfies the stability conditions and is positive. It follows that one unique stable equilibrium exists.

Define $Q_2 = a(\frac{1}{4}(2\delta + r)^2c + 2Q_3) - aQ_1$. Each point (Q_1, Q_2) in the region (1b) can be determined by a $Q_1 \leq \frac{1}{4}(2\delta + r)^2c + 2Q_3$ and an $a \in [0, 1]$. Look at the polynomial in y at the left-hand side of (A.26). This has a local minimum at

$$m = \frac{1}{3}\sqrt{2}\frac{1}{-Q_3}[A + \sqrt{A^2 - \frac{3}{2}Q_3^2}] \tag{B.1}$$

with $A = (1 - \frac{1}{4}a)(\frac{1}{4}(2\delta + r)^2c - Q_1) - \frac{1}{2}aQ_3$, given the definition of Q_2. The unique solution to (A.26) in region (1) is larger than $\frac{1}{2}\sqrt{2}$ (See appendix A). It must be the largest root of the polynomial. If the largest root is real, it must be larger than the local minimum, m. That follows from (A.27), because $\frac{1}{2} \leq cos(\frac{\psi}{3}) \leq 1$. If it can be proved that m satisfies the stability criterium, that is (see (A.38)) $b_4 < m$, then the unique solution is always stable.

The derivative of m with respect to Q_1 can be shown to be negative in the region concerned (region 1b). For fixed a, m therefore reaches its minimum value, m^*, in region (1b) for $Q_1 = \frac{1}{4}(2\delta + r)^2c + 2Q_3$. It is given by (B.1) with $A = -2Q_3$ and equals $m^* = \frac{2}{3}\sqrt{2} + \frac{1}{3}\sqrt{5}$.

The derivative of b_4 with respect to Q_1 can be shown to be positive in the region concerned. Hence, b_4

reaches its maximum value in (1b) for $Q_1 = \frac{1}{4}(2\delta + r)^2 c + 2Q_3$. It is given by

$$b_4^* = \frac{1}{2}\sqrt{2} + \frac{1}{8}\sqrt{2}\frac{r^2c^2}{B} + \frac{1}{4}\frac{rc}{B}\sqrt{6B + \frac{1}{2}r^2c^2} \tag{B.2}$$

with $B = -\frac{1}{4}r^2c^2 - 2Q_3$. Note that for $Q_3 < -\frac{1}{8}(2\delta + r)^2 c$, $B > 0$.
To prove that $b_4 < m$ in region (1b), it is sufficient to prove that $b_4^* < m^*$. This is equivalent to:

$$3r\sqrt{-24Q_3c - 2r^2c^2} < -8Q_3 - 4r^2c + \sqrt{10}(-r^2c - 8Q_3) \tag{B.3}$$

Straightforward calculations show that this inequality is satisfied for $2Q_3 < -r^2c$. This concludes the proof of proposition 1.

C Exclusion of multiple equilibria under standards

From (15) follows that in an equilibrium (K, K), $\Pi^i_{ii} = \beta(\beta-1)p_0\sqrt{\bar{e}}K^{\beta-2} - \beta(4\beta - 2 + \alpha\beta - \alpha)\bar{e}K^{2\beta-2}$, $\Pi^i_{jj} = -\alpha\beta(\beta-1)\bar{e}K^{2\beta-2}$ and $\Pi^i_{ij} = -\alpha\beta^2\bar{e}K^{2\beta-2}$. Add these together to see that

$$po(1-\beta) + \sqrt{\bar{e}}K^{\beta}[4\beta + \alpha\beta - 2 - \alpha] + \alpha(\beta-1)\sqrt{\bar{e}}K^{\beta} - 2\alpha\beta\sqrt{\bar{e}}K^{\beta} > 0 \tag{C.1}$$

is a sufficient condition for

$$\Pi^i_{jj} < \frac{1}{4}(2\delta + r)^2 c + 2\Pi^i_{ij} - \Pi^i_{ii}. \tag{C.2}$$

Condition (C.1) is satisfied if $4\beta > 2(1+\alpha)$ or $\sqrt{\bar{e}}K^{\beta} < -p_0\frac{(1-\beta)}{4\beta - 2(1+\alpha)}$. Since $\alpha \leq 1$, a sufficient condition for this is $\sqrt{\bar{e}}K^{\beta} < \frac{1}{4}p_0$. This requires β and $\bar{e}$ to be sufficiently small. If (C.2) holds, region (4) is excluded.

D Sign of $\mathbf{P}_1$

Use definition A.21 and set $P_1 < 0$. It follows that this requires

$$|\tan\phi| > \frac{\frac{1}{2}(2\delta + r)c}{\sqrt{-Q_1 c}} \tag{D.1}$$

given that $-\Pi < \phi < 0$. If δ, r and c are small enough, so that $Q_1 < -\frac{1}{2}(2\delta + r)^2 c$, the right-hand side of this inequality is smaller than $\frac{1}{2}\sqrt{2}$. It follows that any stable solution (which by necessity is characterized by $|\tan\phi| > \frac{1}{2}\sqrt{2}$) satisfies inequality (D.1) and hence has a negative P_1.

Chapter 4

Emission permits and investment in emission reduction

This chapter takes a first step towards the inclusion of another environmental policy instrument, transferable permits, into the duopoly model described in the foregoing chapters. A firms' optimal investment behaviour is analysed over time, when that firm is subject to a system of emission permits with given end time. Firms may bank and trade their permits.

It is shown that firms invest in abatement in earlier periods, advancing compliance with future environmental standards, when they are allowed intertemporal flexibility in the allocation of their emissions. Therefore, granting firms individual flexibility in the form of 'emissions banking' enhances the efficiency of marketable permits and may result in substantial cost savings.

The firm by assumption faces a two step emission standard with strict requirements at the end of the program. The firm's optimal trajectory under a pure banking program is compared to optimal trajectories under command-and-control and under a system of emission taxes. Finally, the dynamics of an emission trading program are analysed, in which permits are available in a perfectly competitive market but do not last forever.

1 Introduction

Environmental regulation through a system of transferable pollution permits implies that scarcity on a well defined new good, namely the services of the environment, is created (Dales, 1968). The regulator establishes an overall standard and distributes emission permits among firms. Firms need to have sufficient permits if they want to emit pollution into the environment. The firms are allowed to trade their permits in a well-defined market. It has been demonstrated that this instrument is a cost-effective strategy for addressing pollution problems within a static once-and-for-all setting (Montgomery, 1972; Tietenberg, 1985; Baumol and Oates, 1988).

Actual environmental policy designs take specific characteristics of the environmental problem into account. That results in designs that may be different from theoretical models. The specification of targets often takes the form of steps. For example, the European Union requires its member countries to reduce sulphur emissions in three phases of five years each. At the end of each phase, all countries should comply with an emission reduction based on their baseline emissions of 1980.[1] Another example is the U.S. Acid Rain Program.[2] This program requires electric utilities to reduce their rate of emissions for several pollutants in two reduction phases (1990-1995 and 1995-2000). An emission permits market within the electricity industry should in principle increase a utility's flexibility in meeting the emission standard. Since a large market was created and serious emission reductions were required, this led to important expectations on the program. But a recent evaluation of the Acid Rain Program shows negligible trade among firms (Burtraw, 1996). There is some trade in the market as outsiders are buying permits. However, most firms are reluctant to sell the permits that they do not yet need. The Lead in Gasoline Program[3] included a stepwise reduction of standards as well. This program required refineries to gradually reduce, and finally eliminate, the lead content in their gasoline production. The program allowed for the trade in reduction quotas among refineries as well as for banking of quotas. Unused quotas could be saved during some period and then be used to justify emissions in a later period.

From these examples, some characteristics stand out. First, global and local pollution targets are often fixed in several more or less explicit phases of emission reduction requirements. Second, regulators often require certain individual environmental standards to be met at some future point in time. They create a period of adjustment for polluters, but they also want that each individual polluter satisfies the requirements at the end of that period. Finally, real world transferable permit systems often include the option to bank permits. Polluters are allowed to save current surplus allowances for their own future use or for future sale to other polluters. Data on the emission permit market in Los Angeles (Foster and Hahn, 1995) show that banked trades represent about 30% of the total number of trades through 1990. Banked trades mean that the seller of the permits has previously put his permits in a specially established permits bank.

Due to the timing aspect, the banking of permits cannot be studied in a static model. It is therefore interesting to consider dynamic models of a permit program, where banking can be included in the model. Cronshaw and Kruse (1996) analyses the dynamic properties of a market for permits within a discrete time model with finite horizon. They show that, under perfect information and with both trading and banking allowed for, an efficient permit price pattern exists. Moreover, without further restrictions, in equilibrium firms bank only if permit prices rise at the rate of interest and marginal abatement costs stay constant over time. In case permit prices rise less than the rate of interest, firms prefer to sell their permits and save the revenues at a bank. A market equilibrium in which prices rise faster than the interest rate cannot exist. Then all firms would prefer to bank permits at an infinite rate. In case abatement

[1] See the Second (European) Sulphur Protocol (SSP), 1994.

[2] See for a description Kete, 1992 or Burtraw, 1996. See also the Clean Air Act Amendment (US, 1991).

[3] This program ran from 1983 to 1986. See EPA, 1985. See for a description Nussbaum, 1992.

costs decrease over time, firms prefer to use permits and delay abatement. Cronshaw and Kruse model abatement as a recurrent cost. Therefore, instead of investment in abatement technology, the model assumes abatement techniques like input substitution, where variable costs are the most important component.

Rubin (1996) is another model of a dynamic permit market. This paper allows for both emissions banking and emissions borrowing. Rubin proves existence and efficiency of a market equilibrium in a continuous-time model. Like Cronshaw and Kruse, the model assumes that abatement consists of recurrent costs, and does not allow for investment in emission-reduction technology.

Investment in abatement technology under marketable permits is analysed in Kort (1996). This paper compares the optimal behaviour of a firm under taxes and permits. The permits give right to a given amount of emissions per time period and have infinite validity. That implies that permit holdings should be interpreted as a kind of capital stock, which 'produces' emission rights. In line with this interpretation, permits are assumed to depreciate, which implies that the regulator gradually strengthens the environmental requirements. The 'rental cost of a permit' then equals the price of permits times the sum of interest and depreciation rate. This is what the firm must pay for the right to emit a unit of emissions at some point in time. It is shown that in steady-state equilibrium, a uniform tax on emissions is equivalent to a permit system if permit prices are such that the tax equals the 'rental costs of a permit'. In that case, to the individual firm taxes and permits are equivalent. An infinite time horizon is considered and there are no requirements on the firm's final abatement technology.

The combination of overall with individual emission standards, which characterizes for example the Acid Rain Program, is an aspect that has not been explicitly addressed in the literature. If permits are not valid forever, the regulator implicitly requires each firm to actually attain some level of abatement at the end of the program, whatever the net position of the firm in the permit market during the program. Hence, at the end of the program, each individual firm must meet the standard. This raises the question what the potential cost savings are from a permit market compared to a fixed emission standard in such a case. Any cost savings from allocative efficiency, which exploit abatement cost differences among firms, can be only temporal. At the end of the program all firms should satisfy individual requirements, and no firm can always choose to buy permits and refrain totally from investment in abatement. That means that at the end of the program, potential allocative efficiency gains are not exploited. Cost savings may result from differences between the firms in the preferred timing of their investment. If some firms prefer to delay their investment, while others prefer early investment, then a temporal allocative efficiency gain exists, which can be exploited if these firms can trade permits. The individual reallocation of emissions over time through banking may also result in cost savings.

In this chapter we focus on the role of a firm's investment planning under a flexible environmental policy, that allows for the dynamic reallocation of emissions. The program has a finite end time and at the end of the program the firm is required to satisfy an individual emission standard. Finite time horizons come close to current environmental programs and introduce interesting conditions in the specification of dynamic models for permits. The performance and effectiveness of intertemporal trading are evaluated for a program that wants to achieve an emissions objective by each individual firm (a limit on its rate of emission) at the end time. Whatever is the position of the firm during each period, it must comply with that objective in the last year (when permits are not valid any more). This definition implies the combination of two instruments, tradeable permits and an individual emission standard. The 'step' implementation of abatement targets is another characteristic of the model analysed in this chapter.

Summarizing, this chapter is motivated by three aspects of actual marketable permit markets, namely step implementation of targets, finite validity of permits and individual end-time requirements. Our aim is to evaluate the costs and benefits of the intertemporal distribution of abatement targets. What happens to the overall emission standard and to firms' intertemporal distribution of investment in abatement when intertemporal reallocation of emissions is possible? To this purpose, we develop a model and evaluate the impact on a firm's optimal investment behaviour of permit banking and trade. The dynamics of banking and emissions trading are analysed when the firm faces a two-step emission standard. The model differs from other dynamic permit models, because investment in emission reduction is explicitly modelled.

For banking policy, the optimal distribution of emissions over time is characterized from the point of view of the individual firm. The firm must decide when to introduce abatement. Suppose that the regulator has the sum of all emissions during the whole period $\int_0^T S(t)dt$ as its objective, instead of $S(t)$ for each t. For stock pollutants, such as greenhouse gases and chlorofluorocarbons, that is a reasonable assumption. For this type of pollutants accumulated emissions are important. The distribution of emissions over time matters less in terms of environmental impact than the total amount of emissions during the whole period. In that case, the regulator may choose to leave the allocation of emissions over time to the firms.[4]

For comparison, command-and-control regulation is considered, that requires the firm to satisfy an individual standard at each moment in time. The command-and-control model is a deterministic version of Hartl (1992), who analyses the investment behaviour of a firm in a stochastic environment, in which the introduction date of a stricter standard and the requirements of that standard are uncertain. When the introduction date is the start of the second phase of the program and the standard is set equal to the emission limit of the second phase, the outcome can serve as an upper bound for the firm's investment costs under a banking system. A lower bound is provided by a system that leaves the firm complete freedom to reach the end-point requirement. Given convex investment costs, banking induces the firm to over-comply with environmental standards in earlier periods, compared to command and control. Under banking, the firm may thus spread its adjustment to future tighter policies more evenly over time.

The chapter is organized as follows. Section 2 formulates the environmental policy scenario and derives the conditions under which an individual firm finds banking advantageous. The costs that a firm may save by individual intertemporal emission reallocation compared to command and control are derived. We characterize the investment decisions of a firm that

[4]When other environmental problems are closely related to the pollutant, then this may result in undesirable peak-loads. For instance, the use of fossil fuels not only causes emissions of CO_2, which contributes to the accumulation of greenhouse gasses in the atmosphere, but also emissions of particles that cause other types of environmental damage.

has the flexibility to choose the time distribution of its emissions during the program and, therefore, its investment in emissions abatement, provided that it has sufficient permits to cover its emissions. Section 2.5 compares the firm's investment under permit banking with its investment under emission taxes. A condition is presented for taxes to result in the same optimal behaviour as permit banking. In section 3.1, we model investment under a pure emission trading program. The time horizon of the program is finite and firms must satisfy individual end-point requirements. Then in section 3.2 a system with banking and trading is considered. For both variations, existence of a market equilibrium is derived. Finally section 4 concludes.

2 Banking as an intertemporal transfer of emissions

2.1 A formal model for permit banking

Consider a firm which is faced with a system of tradeable permits during the period $[0, T]$. The firm is assumed to have minimization of abatement costs as an objective.[5] In order to focus on time flexibility, it is assumed that the firm is allowed only to bank the permits: it can save permits for its own later use, but it cannot trade its emission surplus with other polluters. This deliberately limits the use of permits to capture only the effects of time flexibility. In the next section the model is extended and includes trade in permits.

The permit program has the following characteristics: the regulator introduces a permit system at time 0. A permit is defined as a license to emit some fixed amount of pollution at any time during the program, say x tons of SO_2. The regulator announces to the firm that in the final period T it has to comply with a specific (stricter) emission standard $\ell(T)$. Hence, at some point in time each firm must technically adapt to meet environmental requirements. Permit banking allows the firm to adjust at its own speed.

The firm's production process causes emissions, $E(t)$. Emissions can be decreased by investment in abatement capital. Let abatement capital be denoted by K, then emissions at time t are given by $E(K(t))$. It is assumed that a higher abatement capital stock results in lower emissions, $E' < 0$, and that it is more costly in terms of capital to decrease emissions when these are already low, $E'' \geq 0$. Given the regulation and the assumption of cost minimization, the firm will try to minimize the amount of money which it has to spend on abatement. For a certain standard ℓ, we can invert $E(.)$ to find $K = E^{-1}(\ell)$, the amount of capital which is required to satisfy this standard. This amount is denoted by $k(\ell)$.

At time $t = 0$, the regulator announces what standard $\ell(t)$ will hold during the period $[0, T]$. The standard becomes stricter over time, therefore $\ell(t)$ is a decreasing function of time. Specifically the following scheme is assumed:

$$\ell(t) = \ell_1 \quad t \in [0, s] \tag{1}$$

[5] Think of a public utility, for which it is not always appropriate to describe the objective to be profit maximization.

$$\ell(t) = \ell_2 \quad t \in (s, T] \tag{2}$$

with $\ell_1 > \ell_2$

The firm is assumed to have enough abatement capital at time 0 to satisfy the standard $\ell(0) = \ell_1$:

$$K(0) = k(\ell_1). \tag{3}$$

After this period the emission standard is tightened to the final level. This way of standard setting is almost equivalent to the Acid Rain Program (see for a description of this program Kete (1992)). The announcement of the Acid Rain Program induced firms to anticipate the first period standard. When the program started in 1995 most firms already complied with the first period limit (Burtraw, 1996).

The firm can create permits and save these in a bank. The firm creates permits if it emits less than the standard $\ell(t)$. It can bank these and use them later. The firm is allowed to emit more than the standard if it has previously banked permits to cover the additional emissions. Banking permits increases the stock of permits at the bank, $A(t)$. This stock neither depreciates nor earns interest.[6]

Most regulators are vague about the continuation of the program after time T. For example, the Acid Rain Program does not specify what will happen with existing permits or the standard after the year 2005 (see Clean Air Act Amendment (US, 1991)). The Second (European) Sulphur Protocol (SSP) specifies new international targets for SO_2, but leaves the implementation of future stringent standards open (see SSP, 1994). In the Lead-in-gasoline program, the standard $\ell(T)$ remained valid, while all previously banked permits lost their value (see Nussbaum, 1992). In our model, it is assumed that permits loose their validity when the program ends. For abatement capital a linear scrap value function $S(K) = v_k K$ is introduced to reflect the expectations of the firm on future regulations. When the firm expects stricter future regulation, it attaches a positive scrap value to K.

The amount of permits created and banked (or withdrawn from the bank and used) at some time t, is given by the difference between actual emissions and the standard. If the firm emits more than the standard, it withdraws permits from the bank. If it emits less than the standard, it adds permits to the bank account. The increase in the stock of banked permits is given by:

$$\dot{A}(t) = \ell(t) - E(K(t)) \tag{4}$$

The bank requires firms to hold a non-negative stock of permits, that is loans are not provided:

$$A(t) \geq 0 \tag{5}$$

Furthermore, the firm is assumed to start with no banked permits at time 0:

$$A(0) = 0 \tag{6}$$

[6] In the Emissions Trading Program (1981) in the United States, banked permits could depreciate according to required environmental quality improvements in the areas implemented (Padron, 1991).

Abatement capital can be accumulated according to a standard capital accumulation function:

$$\dot{K}(t) = I(t) - \delta K(t) \tag{7}$$

where δ is the rate of depreciation and $I(t)$ is gross investment in abatement capital. If the firm invests it is subject to acquisition and adjustment costs. These costs are given by the investment cost function $C(I)$. It is assumed that $C(0) = 0$, $C' \geq 0$ and $C'' > 0$. The firm determines its investment in abatement and the permits it banks at each time t to minimize its discounted stream of investment costs minus the scrap value at the final time T.
At time T, when the program ends, the firm is required to satisfy the standard without using permits: $E(K(T)) \leq \ell(T) = \ell_2$. The permit system is introduced to allow the firm to choose its own time path for accumulation of abatement capital and obeyance of a stricter standard, ℓ_2. It is intermediate between a system with complete time flexibility, in which the firm is only required to satisfy ℓ_2 at the final time T and a system with no time flexibility, in which the firm must satisfy the standard $\ell(t)$ at any time.
Summarizing, the firm has to solve the following constrained dynamic optimization problem: minimize the present value of investment costs during the whole program minus the scrap value of abatement capital in the last period,[7]

$$\min_{I \geq 0} \left\{ \int_0^T e^{-rt} C(I) dt - v_k K(T) e^{-rT} \right\} \tag{8}$$

subject to the following constraints,

$$\dot{A} = \ell(t) - E(K) \tag{9}$$

$$\dot{K} = I - \delta K \tag{10}$$

$$A \geq 0 \tag{11}$$

$$A(0) = 0 \tag{12}$$

$$K(0) = k(\ell_1) \tag{13}$$

$$E(K(T)) \leq \ell_2 \tag{14}$$

When environmental policy is given by the scheme of standards (1) to (2), the standard $\ell(t)$ is discontinuous at s. This implies that the optimization problem does not satisfy the standard continuity requirements (cf. Seierstad and Sydsæter, 1987, p.73). However, the weaker regularity conditions mentioned in the same volume are met (note 5.11, Seierstad and Sydsæter, 1987, p.348), so that the maximum principle can still be applied. The first-order optimality conditions can be rewritten in the following form (see appendix A):

$$\lambda_1 \leq C'(I) \quad I \geq 0 \quad I[C'(I) - \lambda_1] = 0 \tag{15}$$

[7]Here and in the sequel time t is often suppressed to shorten notation.

$$\lambda_2 = \frac{\dot{\lambda}_1 - (r+\delta)\lambda_1}{E'(K)} \tag{16}$$

$$\mu = r\lambda_2 - \dot{\lambda}_2 \tag{17}$$

$$\mu \geq 0 \;\; A \geq 0 \;\; \mu A = 0. \tag{18}$$

Furthermore the terminal conditions,

$$\lambda_1(T) \geq v_K \;;\; K(T) \geq k(\ell_2) \;;\; (K(T) - k(\ell_2))(\lambda_1(T) - v_K) = 0 \tag{19}$$

$$\lambda_2(T) \geq v_a \;;\; A(T) \geq 0 \;;\; A(T)(\lambda_2(T) - v_a) = 0 \tag{20}$$

must hold. Since it is an optimal control problem with a pure state constraint, jumps in λ_2 cannot be excluded. Equation (15) refers to optimal investment in abatement capital. Except for corner solutions, the shadow value of an additional unit of abatement capital, λ_1, must equal marginal investment costs. The second equation, (16), links the shadow value of a permit at the bank, λ_2, to the firm's abatement. The more the firm has invested in abatement capital, the costlier it is in terms of capital to reduce emissions even further (since $E'(K) \leq 0$) and, hence, the higher the value of a permit at the bank. Equations (17) and (18) link the stock of permits at the bank to its shadow value. If this shadow value is decreasing in real terms (so $r\lambda_2 - \dot{\lambda}_2 > 0$), then from (18) it follows that $A = 0$, that is it is not optimal to hold a positive stock of permits. If an optimal solution is characterized by positive amounts of permits at the bank, the permits' shadow value must grow at exactly the interest rate.
Along an optimal path the shadow value of a permit at the bank will never grow at a rate higher than the interest rate. This follows from (18). If the shadow price would grow faster, the firm would want to bank an infinite number of permits, hence no optimal solution would exist. A second characteristic of the shadow value of permits at the bank along optimal investment paths, is given in the following proposition:

Proposition 4.1 *An additional permit at the bank is never valued negatively by the firm, or formally,*

$$\lambda_2(t) \geq 0 \;\; \forall t \in [0, T]. \tag{21}$$

Furthermore if $\lambda_2 = 0$ for some t_1, then $\lambda_2 = 0$ for all $t \geq t_1$.

It is reasonable that the shadow value of a permit is nonnegative, since the firm can choose to use the permit at any time. The proof to this proposition is as follows: Assume that $\lambda_2 \geq 0$ does not hold for all t, so that $\lambda_2(t_1) < 0$ for some $t_1 \in [0, T]$. Then, from equation (17) and (18) it follows that $\dot{\lambda}_2 < 0$ for all $t \geq t_1$. This contradicts the terminal condition $\lambda_2(T) \geq 0$. To prove the second part of the proposition, let $\lambda_2(t_1) = 0$ for some $t_1 \in [0, T]$. Then, from (17) $\dot{\lambda}_2(t_1) = -\mu \leq 0$. It was just derived that $\lambda_2 \geq 0$. Hence, $\dot{\lambda}_2(t_1) = 0$. The same reasoning applies for all $t \geq t_1$ and results in $\lambda_2 = 0$ for all $t \geq t_1$.

From proposition 4.1 and equation (16) it follows that along an optimal path, the shadow value of abatement capital grows at most at the rate $(r + \delta)$. If the shadow value of would grow faster, firms would prefer to borrow permits and delay investment. That would, given (16), mean a negative shadow value for permits at the bank. This is excluded along an optimal path by proposition 4.1 and is a logical consequence of the model set up, that excludes borrowing.

Without any further analysis of the model, it can be seen that this system of banking is always advantageous to the firm compared to a command-and-control policy. Consider a firm that is not allowed to bank permits, but is subject to a command-and-control standard: it has to satisfy the standard $\ell(t)$ at any moment. In terms of the permit system, the firm is not allowed to use any permits. This is equivalent to adding the constraint

$$E(K(t)) \leq \ell(t) \ \forall t \tag{22}$$

to the cost-minimization problem above. First-order conditions for this optimization problem and its full formal specification are given in appendix B. Allowing for the use of permits implies that the constraint (22) is removed from the firm's decision problem, who must then solve the problem stated in (8) to (14). The optimal solution with permits will be at least as good as the optimal solution under the command-and-control standard. It is very likely to be better, implying cost savings, since the firm can smooth its investment.

2.2 Cost savings in a banking system

This section analyses the cost savings that can be obtained by the system of banking (8) to (14) compared to a command-and-control system. That is, positive cost savings result if the banking system performs better than the command-and-control standard. To analyse the cost savings, first an upper and a lower bound to these cost savings are derived. For simplicity, it is assumed in this section that capital above the minimum requirement at time T has no value, so that $v_k = 0$. The scenario with $v_k > 0$ is shortly addressed at the end of this section.

A natural lower limit to the cost savings is 0. When no cost savings are obtained, the permit system performs no better than the command-and-control policy. In that case, the firm follows its command-and-control trajectory (the solution to (8) to (14) and (22)), even if under the permit system it does not need to satisfy the constraint (22). Denote the optimal trajectory of the capital stock for the command-and-control standard by $K^{cac}(t)$ and optimal investment by $I^{cac}(t)$.

An upper limit to the cost savings that can be obtained by the firm is given by the cost savings for a firm that is allowed to satisfy $E(K(T)) \leq \ell_2$ without any further requirements on permits, rather than required to satisfy $E(K(s)) \leq \ell_2$ (as it would be under the command-and-control policy just described). This firm then has to solve the optimal control problem defined by equations (8), (10), (13) and (14). It is a deterministic version of the problem discussed in Hartl (1992). We denote the optimal trajectories for this optimal control problem by $K^F(t)$

and $I^F(t)$. First-order conditions can be simplified to the system (see Hartl, 1992):

$$\dot{I} = \frac{C'(I)}{C''(I)}(r+\delta) \tag{23}$$

$$\dot{K} = I - \delta K; \;\; K(0) = k(\ell_1); \;\; K(T) = k(\ell_2) \tag{24}$$

which can be solved for optimal investment, $I^F(t)$, with capital trajectory $K^F(t)$. Both investment rate and capital stock increase over time along this optimal path. Compared to the command-and-control policy, the cost savings to be obtained along this trajectory are given by:

$$\int_0^T e^{-rt} C(I^{cac})dt - \int_0^T e^{-rt} C(I^F)dt \tag{25}$$

For $C(I) = \frac{1}{2}cI^2$, it can be derived that potential cost savings are larger, the larger is c, that is the more convex are investment costs. To see this, compute the optimal investment paths for $C(I) = \frac{1}{2}cI^2$ and the value of the expression (25). It is shown in appendix C that for $C(I) = \frac{1}{2}cI^2$, both optimal investment paths, I^{cac} and I^F, are independent of c. It follows then easily that the derivative of expression (25) with respect to c equals $\frac{1}{2}\int_0^T e^{-rt}[(I^{cac})^2 - (I^F)^2]dt$ and is positive.

Returning to our model, let a firm follow the trajectory $I^F(t)$ and assume that it can obey the permit requirements. That is, assume that the permits that can be saved under the lax standard, ℓ_1, which holds during the period $[0, s]$, are sufficient to cover excess emissions in the second period $[s, T]$. If these assumptions are true, then permit requirements are not a binding constraint and it is feasible for the firm to choose I^F with capital trajectory K^F also in a permit-banking system. Moreover, this trajectory is an optimal solution. The trajectory $K^F(t)$ is optimal when permits are absent. If this path is still feasible with permits, because permits at the bank are not a binding constraint in the period $[s, T]$, then it is also optimal. Formally, from $A(T) > 0$ and the terminal conditions, it can be seen that $\lambda_2(T) = \mu = 0$. If that is used and inserted into the first-order conditions, they can be rewritten to the conditions (23) and (24). A formal statement of the condition that enough permits should be saved in the first period leads to the following proposition:

Proposition 4.2 *Let I^F and K^F denote optimal investment and capital that result from (23) and (24). Define*

$$A^F = \int_0^s [\ell_1 - E(K^F)]dt + \int_s^T [\ell_2 - E(K^F)]dt \tag{26}$$

When $A^F \geq 0$, the firm can realize the maximal cost savings compared to a command-and-control problem if it chooses the investment path $I^F(t)$ for all t.

Cost savings are then given by (25).

2.3 Characterization of investment under banking

For a further characterization of investment under permits, it is useful to consider the two periods $[0, s]$ and $[s, T]$ separately. The first-order conditions that must hold in both periods allow for two potentially optimal paths, which we call $(p1)$ and $(p2)$ respectively and which differ by the value of μ. When $\mu > 0$, conditions (15) to (18), (9) and (10), define a path $(p1)$ given by: $A(t) = 0$, $\dot{A}(t) = 0$, $K(t) = k(\ell(t))$, $\dot{K}(t) = 0$, $I(t) = \delta k(\ell(t))$, $\lambda_1(t) = C'(I(t))$, $\dot{\lambda}_1(t) = 0$, $\lambda_2(t) = -\frac{(r+\delta)\lambda_1(t)}{E'(K(t))}$, $\dot{\lambda}_2(t) = 0$, and $\mu(t) = r\lambda_2(t)$. When $\mu = 0$, from (17) it follows that $\lambda_2(t) = \gamma e^{rt}$ for some $\gamma \geq 0$. This path, $(p2)$, is then defined by (15), (16), (9) and (10), with $\lambda_2(t) = \gamma e^{rt}$.

The period $[s, T]$
During the period $[s, T]$, for a given $K(s)$ and $A(s)$, the firm must invest enough to satisfy the strict standard ℓ_2 before the end of the program while it is not allowed to use more permits than it has available.
Since the rate of investment is not bounded by some maximum, all combinations of $A(s) > 0$ and $K(s) \geq 0$ are feasible, that is they allow for a solution to the optimal control problem in $[s, T]$. The set of feasible combinations of $A(s)$ and $K(s)$ can be divided in two sets. The first set, (set1), consists of combinations for which permits are a binding constraint, so that the firm must adjust its investment in order not to use too much permits. The second set, (set2), contains combinations for which $A(s)$ is high enough, so that permits are not a binding constraint.
Define by $K^f(t; K(s))$ the optimal path chosen by a firm not restricted by permits that must satisfy $K(T) \geq K(\ell_2)$ and starts with $K(s)$ at time s.[8] For large enough $A(s)$, the firm can choose this path and have sufficient permits to cover its emissions until time T.
The boundary between the two sets is given by a function denoted $A^m(K(s)) = -\int_s^T [\ell_2 - E(K^f(t; K(s)))]dt$. All $A(s) < A^m(K(s))$ belong to (set1) and all $A(s) \geq A^m(K(s))$ belong to (set2). The higher $K(s)$, the lower the minimum amount of permits needed for the firm to choose its unrestricted path of investment, K^f. Hence A^m is decreasing in $K(s)$.
For $A(s)$ in (set1), available permits are a real restriction to the firm in the second period. The firm must adjust its investment rate, so that it needs less permits than it would have needed, had it chosen $K^f(t; K(s))$. It can be proved that along an optimal path, the firm follows $(p2)$ for a period $[s, s_1)$ and then switches to $(p1)$ for the period $[s_1, T]$, where $s_1 \in (s, T]$ (see appendix D). Denote the optimal trajectory of capital by $K^p(t; K(s))$.

The period $[0, s]$
Now consider the first period, $[0, s]$. In this period, the firm has to invest to arrive at a given $K(s)$ and $A(s)$ at time s. Again, the set of possible combinations $A(s)$ and $K(s)$ can be divided in two subsets. The first set, (set3), contains the combinations that require the firm to adjust its investment rate to save enough permits. The second set, (set4), consists of combinations that can be reached if the firm invests at a rate that is optimal to go from $K(0)$ to $K(s)$, without special consideration to save enough permits.

[8] The path $K^F(t)$ is a special case of these paths, with a specific value for $K(s)$.

Denote by $K^f(t; K(s))$ the optimal path for a firm that has to satisfy $K(s)$ at time s, but is not constrained by permits. Let the permits saved by this firm be given by $A^n(K(s)) = \int_0^s [\ell_1 - E(K^f(t; K(s)))] dt$. Then (set4) consists of combinations $K(s)$ and $A(s)$ with $A(s) \leq A^n(K(s))$. In that case, $A(s)$ is so low that a firm that invests according to $K^f(t)$ saves enough permits to satisfy $A(s)$. The combinations in (set3) are defined by $A(s) > A^n(K(s))$.
In (set3) more permits than $A^n(K(s))$ are required and the firm must adjust its path to $K^p(t; K(s))$. Along such a path, the firm starts with $(p1)$ for $[0, s_2)$ and switches to $(p2)$ for $[s_2, s]$, where $s_2 \in [0, s)$ (see appendix D). Along such a path, permits are a binding constraint, so that λ_2 is positive.

The whole period $[0, T]$
Combining the two periods, four possible cases exist:

a) $0 < A(s) \leq A^n(K(s))$ and $0 < A(s) \leq A^m(K(s))$; the firm can follow $K^f(t; K(s))$ in $[0, s]$, but has to adjust to $K^p(t; K(s))$ in $[s, T]$.

b) $0 < A(s) \leq A^n(K(s))$ and $A(s) \geq A^m(K(s))$; the firm can choose $K^f(t; K(s))$ in both periods. This implies that the condition $A^F \geq 0$, is satisfied, so that we are back in the case of proposition 4.2.

c) $A(s) \geq A^n(K(s))$ and $0 < A(s) \leq A^m(K(s))$; the firm must follow $K^p(t; K(s))$ in both periods.

d) $A(s) \geq A^n(K(s))$ and $A(s) \geq A^m(K(s))$; the firm follows $K^p(t; K(s))$ in $[0, s]$ and chooses $K^f(t; K(s))$ in the second period $[s, T]$.

An optimal solution is a cost minimizing combination of $A(s)$ and $K(s)$. Since excess permits are sub-optimal, only the boundaries of the sets in cases (a) and (d) are relevant. That is for (a), only $A(s) = A^n(K(s))$ needs to be considered and for (d), only $A(s) = A^m(K(s))$ contains possibly a cost minimizing combination of $A(s)$ and $K(s)$. Similarly, for (b), the only point of interest is $A(s) = A^n(K(s)) = A^m(K(s))$. Other combinations mean that unnecessary amounts of permits are banked. All the relevant points are part of the boundary of the set in (c). Therefore, it is sufficient to consider (c) and its boundary. A cost-minimizing combination of $K(s)$ and $A(s)$ must be found in this set. Hence, any optimal trajectory is characterized by K^p in both periods, provided that $A^F \leq 0$. It follows that the optimal trajectory can be described as: $(p1)$ for $[s, s_2)$, $(p2)$ for $[s_2, s]$ and for $[s, s_1]$ and $(p1)$ for $(s_1, T]$.
We can compare the optimal path under the permit policy described above to the investment strategy when the firm faces a binding standard The optimal path under a command-and-control policy, $K^{cac}(t)$, is given by: $K^{cac}(t) = K^f(t; K(s))$ in $[0, s]$, with $K(s) = k(\ell_2)$ and $K^{cac}(t) = k(\ell_2)$ in $[s, T]$. It requires an investment of $I^{cac} = I^f(t; k(\ell_2))$ in $[0, s]$ and $I^{cac}(t) = \delta k(\ell_2)$ in $[s, T]$. This path is a feasible solution to the optimization problem (8) and optimal for $K(s) = k(\ell_2)$, but clearly $K(s) = k(\ell_2)$ is not the cost-minimizing choice of $K(s)$. From the large amount of permits left over at $t = T$ it follows that the firm may choose a lower $K(s)$, which leads to a path closer to the cost minimizing path $K^F(t)$ and still satisfy

its permit requirements. This is not surprising; allowing the firm to bank permits improves its flexibility in planning its investment expenditures. Even in the highly simplified setting under consideration, banking permits results in cost savings.

2.4 The influence of scrap values

All the above reasoning hold for the case $v_k = 0$, in which it is optimal that $E(K(T)) = \ell_2$. Investment in more capital is suboptimal. If v_k is high, it may be expected that the firm will continue to invest in abatement capital even if it satisfies the new stricter standard. Therefore, the shape of the optimal path depends on the definition of environmental policy after the program has stopped. It is easy to derive that for the upper and lower boundary cases (that is, the paths denoted by K^{cac} and K^F), for the simple convex investment cost function $C(I) = \frac{1}{2}cI^2$, boundary values $v_k^{*,cac}$ and $v_k^{*,F}$ exist, which determine the investment behaviour of the firm. Only for scrap values higher than this boundary value, it is optimal for the firm to invest more than what is sufficient to end with exactly $k(\ell_2)$. The optimal investment path is affected by the value of v_k then. For scrap values lower than this boundary value, the firm invests just sufficiently to end with a capital stock of $k(\ell_2)$. In that case, the optimal investment path is independent of v_k. Since the optimal investment path for permit banking lies somewhere in between the paths I^{cac} and I^F, a boundary scrap value for this path will also be in between the boundary scrap values for command-and-control and the upper boundary path. The details for the behaviour of the firm in case of permit banking and positive scrap values, for the given investment cost function and for more general cost functions, are left for further research.

2.5 An emission tax for a given time interval

It is in principle possible for a regulator with perfect information on firms' abatement costs to set up a system of emission taxes that leads the firm to the same time path of investment as it would choose under the permit program described above. The regulator has to compute the optimal investment path and set a tax on emissions equal to the shadow value of banked permits. In the case of a system of emission taxes, the firm has to pay a tax, τ, for every unit of emissions. Thus, it also pays explicitly for the emissions that are below the standard. This is in contrast with the system of permits just described, in which the firm only pays an implicit opportunity cost. The optimization problem that the firm has to solve under taxes is to minimize the discounted stream of abatement costs plus the emission tax minus the scrap value of abatement capital (which represents the firm's expectations on policy developments after T):

$$\min_{I \geq 0} \left\{ \int_0^T e^{-rt}[\tau E(K) + C(I)]dt - [v_k K(T)]e^{-rT} \right\} \tag{27}$$

subject to constraints (10), (13) and (14). Optimality conditions for the firm are:

$$C'(I) \geq \tilde{\lambda}_1 \quad I \geq 0 \quad I[\tilde{\lambda}_1 - C'(I)] = 0 \tag{28}$$

$$\tau = \frac{\dot{\tilde{\lambda}}_1 - (r+\delta)\tilde{\lambda}_1}{E'(K)} \tag{29}$$

with the transversality conditions:

$$\tilde{\lambda}_1(T) \geq v_k; \quad E(K(T)) \leq \ell(T); \quad (\tilde{\lambda}_1(T) - v_k)(E(K(T)) - \ell(T)) = 0 \tag{30}$$

Assuming that an interior solution exists, these conditions are equivalent to those for an optimum under the permit banking system, (15) to (20), when the tax is set such that

$$\tau(t) = \lambda_2(t), \tag{31}$$

where λ_2 is the shadow value of permits from section 2.1. If the regulator sets taxes equal to this rate, the investment path chosen by an optimizing firm subject to these taxes equals that chosen by a firm subject to a system of permit banking as it was described in section 2.1.
It can be concluded that it is possible to arrive at the same results with a tax as with a permit system. But note that this requires the regulator to have perfect knowledge of the cost functions of the firm. Whenever this is not the case it is impossible for the regulator to calculate the correct level of taxes. Moreover, the tax must be adjusted every period to follow the optimal path. For the permit system to reach the optimal path of investment, the regulator only needs to determine the standards, ℓ_1 and ℓ_2. This difference in information requirements is a well known difference between taxes and permits. See for example Baumol and Oates (1988), who point to this difference for tradeable permits and emission taxes in a static model.

3 Trade and banking

3.1 Market equilibrium for tradeable permits

In this section a program that allows only instantaneous trade in permits is considered and banking is ignored. Trade assumes the existence of more than one firm. Denote individual firms by i and assume there are N of them in the market. Assume that N is large, so that firms take permit prices as given. A direct application of a system of tradeable permits in the model from chapters 2 and 3 is not possible. That model should be adjusted to allow for more firms in each country.
The possibility of trade in permits, like banking, allows firms some flexibility in the timing of investments. This flexibility is large when firms in the permit market differ in initial capital stock, investment costs or depreciation rate. For example, consider two firms that only differ in their investment costs: A firm facing investment costs that are almost linear has incentives to delay its investments. Until it has enough capacity to satisfy the standard, the firm buys

permits from a firm with more convex investment costs. The latter firm is interested in a smooth investment path and consequently it sells permits in early periods to the first firm and buys permits in later periods.

Consider a firm i that minimizes the discounted stream of abatement costs over the period $[0, T]$. It is subject to an emission standard $\ell(t)$ during the entire period but it may exceed this standard at some t if it buys sufficient permits from other firms. If the firm emits less than the standard it saves some permits that can be sold in the same period at the permit market. The market price of a permit is given by $p(t)$. Let $y^i(t)$ be the amount of permits the firm buys or sells at the market at time t. Then, y^i is given by:

$$y^i(t) \geq E(K^i(t)) - \ell(t), \tag{32}$$

which is the difference between the firm's emission rate and the standard, which we assume to be the same for each firm. If $y^i > 0$, the firm emits more than the standard and is a net buyer of permits and if $y^i < 0$, the firm is a net seller of permits. The firm invests in abatement capital in order to decrease its emission rate, which increases revenues of permit sales, if the firm is a net seller, or decreases the costs of permit purchases, if the firm is a net buyer. Since market equilibria with excess supply cannot be excluded, it is possible that the firm is not able to sell all the permits that it wants to sell, so that $y^i(t) > E(K^i(t)) - \ell(t)$. Like in section 2, at the end of the period it is required that

$$E(K^i(T)) \leq \ell(T). \tag{33}$$

This implies here that $y^i(T) \leq 0$. Since it has to hold for all firms in the market, $y^i(T) = 0$ for all i, in market equilibrium. All firms must at least comply with the standard at the end of the program, so there is no trade at time T, since no firm needs to buy permits. Cost savings from this policy are derived only from temporary allocative efficiency. Complete allocative efficiency would be reached if firms were allowed to maintain their net position in the permit market beyond T.

It is assumed that permits are not valid after T. Note that the price of the permit at time T reflects the scrap value of a permit in the final period. So, when permits are not expected to be valid after T, $p(T) = 0$. Therefore, the final position on investment in abatement capital of the firm will depend exclusively on its expectations of future tighter emission limits, that is on v_k^i. If the firm does not expect these and $v_k^i = 0$ for all i, it can be expected that $K^i(T) = k(\ell(T))$ and not larger. In permit programs with finite time horizons, when permits explicitly expire in the last period or when vague regulations induce firms to believe so, the incentive to invest more than required by the standard vanishes if no future stricter regulations are expected.

Firms must pay their permit purchases if they are net buyers and receive some benefits if they are net sellers. We consider the standard scheme specified by equations (1) and (2). The optimal control problem to be solved by the firm becomes:

$$\min_{I^i \geq 0} \left\{ \int_0^T e^{-rt}[C^i(I^i) + p(t)(E(K^i) - \ell(t))]dt - v_k^i K^i(T)e^{-rT} \right\} \tag{34}$$

subject to the following constrains:

$$\dot{K}^i = I^i - \delta^i K^i \tag{35}$$

$$K^i(0) = k(\ell_1) \text{ and } E(K^i(T)) \leq \ell_2 \tag{36}$$

Note that here $y^i = E(K^i) - \ell(t)$ has been inserted. For positive prices, $y^i = E(K^i) - \ell$, since it is never optimal for the firm to buy more permits than it needs, or sell less than it can, if prices are positive. If $y^i(t) > E(K^i(t)) - \ell(t)$ for some t, the firm cannot sell all the permits it wants to sell. This implies a market equilibrium with excess supply and zero prices, so that the second term in the objective function is zero and it is innocuous to have $E(K^i) - \ell$, rather than y^i, in the objective function.

The main difference with the banking model is that firms must sell permits generated by excess abatement at the prevailing price. This price is determined as a market equilibrium. For a given market price, first-order conditions for the optimization problem above are derived in appendix E. They can be rewritten to:

$$C^{i\prime}(I^i) \geq \lambda_1^i \quad I \geq 0 \quad I[\lambda_1^i - C^{i\prime}(I^i)] = 0 \tag{37}$$

$$p(t) = \frac{\dot{\lambda}_1^i - \lambda_1^i(r + \delta^i)}{E'(K^i)} \tag{38}$$

with the transversality conditions:

$$\lambda_1^i(T) \geq v_k; \quad E(K^i(T)) \leq \ell_2; \quad (\lambda_1^i(T) - v_k)(E(K^i(T)) - \ell_2) = 0 \tag{39}$$

If an interior solution is assumed, then $\lambda_1^i = C^{i\prime}(I^i)$. Conditions (37) and (38) can be rewritten to find a net present value expression for optimal investment in case of an interior solution:

$$C^{i\prime}(I^i) - C^{i\prime}(I^i(T))e^{(r+\delta^i)(t-T)} = \int_t^T e^{(r+\delta^i)(s-t)}[-p(s)E'(K(s))]ds \tag{40}$$

In words, this condition says that at some time t, for given prices $p(t)$ and future prices $p(s)$, $s \in [t, T]$, investment should be chosen such that the marginal costs of investment at time t (given by $C^{i\prime}(I^i(t))$) compared to the properly discounted marginal costs of investment at the last moment (given by $C^{i\prime}(I^i(T))e^{(r+\delta^i)(t-T)}$) differ by the marginal cost savings from investment now. These marginal cost savings are the marginal payments for permits that can be sold, or need not to be bought. These marginal payments are given by $\int_t^T e^{(r+\delta^i)(s-t)}[-p(s)E'(K(s))]ds$, which is positive, since additional investment at time t rather than at time T implies a higher capital stock and hence lower emissions in the time interval $[t, T]$.

A market equilibrium can be defined, using similar definitions as in Rubin (1996), but adapting them for investment and the individual requirements at time T. Assume that there are N firms in the market and they take permit prices as given.

Definition 4.1 *A price function $p^*(t)$ and investment functions $I^{i*}(t)$ for $i = 1,..,N$ form a* **market equilibrium** *if they satisfy:*
$I^{i*}(t)$ is an optimal solution for firm i to the problem (34) to (36), given prices $p^*(t)$; and

$$\sum_{i=1}^{N}[E(K^{i*}(t)) - \ell(t)] \leq 0; \;\; p^*(t) \geq 0; \;\; p^*(t)\sum_{i=1}^{N}[E(K^{i*}(t)) - \ell(t)] = 0; \; \forall t \in [0,T]$$

It can be shown that such a market equilibrium exists. The proof of this, which is an adapted version of the proof in Rubin (1996), is given in appendix E. Our model differs from Rubin's, because it considers investment and requires individual firms to satisfy end-point requirements. That implies that care should be taken to define the market equilibrium correctly, so that, in contrast to Rubin, excess supply is possible. Equilibrium prices are zero in case of excess supply. An equilibrium at zero prices with excess supply of permits could occur, for example, if all firms are identical. Then in a market equilibrium all firms choose to follow the paths that are optimal under the command-and-control policy, that is invest $I^i = I^{cac}$. This implies that at $t < s$ all firms have accumulated enough capital to satisfy the standard, so that all firms want to sell and no firm wants to buy permits.
Since this program does not allow for banking of permits, all trade must take place instantaneously. Therefore, market equilibria with excess supply need not be confined to the case just mentioned where all firms are equal. Due especially to the constraint (36), a market equilibrium with positive prices requires rather large differences among firms in investment costs or depreciation rates. This constraint implies that all firms start with capital stock $k(\ell_1)$ and all firms must accumulate at least $k(\ell_2)$. For a firm i to be able to sell permits in the time period $[0, s]$ there must be another firm j for which it is optimal to have $I^j < \delta^j K^j(t)$, that is to let its capital stock decrease below its initial stock. For a firm i to be able to buy permits in the time period $[s, T]$, there must be another firm j that has a capital stock such that $E(K^j) < \ell_2$. For this firm j it must be optimal to invest more than is required and sell the extra permits that are created (even if at the end of the program it can no longer sell its permits). If such a firm j does not exist, the market equilibrium has no trade and zero prices. In that case, firms must invest at rate $I^{i,cac}$ and the permit program does not result in cost savings compared to a command-and-control regulation.

3.2 Market equilibrium for permits that can be traded and banked

Adding banking to the model above, the individual firm has to solve the following optimal control problem:

$$\min_{I^i \geq 0, y^i} \left\{ \int_0^T e^{-rt}[C^i(I^i) + p(t)y^i]dt - v_k K^i(T)e^{-rT} \right\} \tag{41}$$

subject to the constraints:

$$\dot{K}^i = I^i - \delta^i K^i \tag{42}$$

$$\dot{A}^i = \ell(t) - E(K^i) + y^i \tag{43}$$

$$A^i \geq 0 \tag{44}$$

$$K(0) = k(\ell_1) \text{ and } E(K(T)) \leq \ell_2 \tag{45}$$

Here, again, $y^i > 0$ implies that the firm buys permits, while $y^i < 0$ implies that the firm sells permits. The main difference with the previous model is that firms have the option to choose between selling excess abatement at the prevailing price or banking it. Since the objective function is linear in y^i, it may for certain prices be optimal for the firm to buy or sell at infinite amounts. Due to the end-point requirements, that will not be a market equilibrium: Buying an infinite amount of permits would not reduce the need to invest in abatement to satisfy the end-point requirements. The maximum amount of permits, $y^i_{max}(t)$, that a firm would need to buy at any time t is given by $y^i_{max}(t) = \int_t^T [E(K^i(s)) - \ell(s)]ds$. This amount could serve to delay further investments until the end time, T, so that it is a natural upper bound for the amount of permits bought by the firm. It is not hard to see that, given convex investment costs, this is not the optimal decision for the firm, even if $p(t) = 0$ for all future t. To find a lower bound, $y^i_{min}(t) < 0$, for the amount of permits bought or, in other words, an upper bound for the amount of permits sold, note that firms cannot reduce their emissions below zero. A minimum to y^i is hence given by $y^i_{min} = \int_0^t [E(K^i(s)) - \ell(s)]ds > -(\ell_1 s + \ell_2(T-s))$: the firm can at most sell all the permits that it has accumulated until time t.
The first-order conditions for the optimization problem above are derived in appendix F and can be rewritten to:

$$C^{i\prime}(I^i) \geq \lambda_1^i \;\; I^i \geq 0 \;\; I^i[\lambda_1^i - C^{i\prime}(I^i)] = 0 \tag{46}$$

$$y^i = \begin{cases} y^i_{min}(t) & \text{if } p(t) > \lambda_2^i \\ y^i_{max}(t) & \text{if } p(t) < \lambda_2^i \\ y^i(t) & \text{if } p(t) = \lambda_2^i \text{ where } y^i_{min}(t) < y^i(t) < y^i_{max}(t) \end{cases} \tag{47}$$

$$\lambda_2^i = \frac{\dot{\lambda}_1^i - \lambda_1^i(r + \delta^i)}{E'(K^i)} \tag{48}$$

$$\mu^i = r\lambda_2^i - \dot{\lambda}_2^i \tag{49}$$

$$\mu^i \geq 0; \;\; A^i \geq 0; \;\; \mu^i A^i = 0 \tag{50}$$

with the transversality conditions:

$$\lambda_1^i(T) \geq v_k; \;\; E(K^i(T)) \leq \ell_2; \;\; (\lambda_1^i(T) - v_k)(E(K^i(T)) - \ell_2) = 0 \tag{51}$$

$$\lambda_2^i(T) \geq 0; \;\; A^i(T)) \leq 0; \;\; \lambda_2^i(T) A^i(T) = 0 \tag{52}$$

It is assumed that when the firm is indifferent between banking or selling its permits, it chooses to bank them.

If an interior solution is assumed, then $\lambda_1^i = C^{i'}(I^i)$ and $\lambda_2^i = p(t)$. The latter implies that in an equilibrium all firms value the permits that they have at the bank equally. Inserting $\lambda_2^i = p(t)$ in equation (49), it can be seen that in market equilibrium $\mu^i = rp(t) - \dot{p}(t)$. If in the market equilibrium, one firm j has a positive stock of permits at the bank, then $\mu^j = 0$. Then, market equilibrium prices must rise at the interest rate and all firms are indifferent between banking their permits or selling them at the permit market and banking the revenues from sales.

If market prices would rise at a rate higher than the interest rate, firms could speculate with permits. They could buy a large amount of permits, bank them and sell them at a higher price later. That cannot be a market equilibrium. It can be seen formally, from (49) and (50) with $\lambda_2^i = p(t)$ inserted, that along individual optimal paths which are interior solutions, $\mu^i \geq 0$, so that $\dot{p}(t) \leq rp(t)$ must hold. For prices that rise at a higher rate, it is for all firms an optimal choice if they choose $y^i = y^i_{max}$, which implies all firms demand permits. This cannot be a market equilibrium because there is excess demand.

If equilibrium prices rise at a rate lower than the interest rate, firms are better off if they reduce their banked permits, sell their permits at the market and save the revenues in a normal bank account. Formally, from (49) and (50) it follows that $\mu^i > 0$ if prices rise at a rate lower than the interest rate, so that $A^i = 0$. In a market equilibrium where prices rise at a low rate, no firm has a positive stock of permits at the bank. Hence, in a market equilibrium where firms bank permits, prices rise at the interest rate.

A market equilibrium can be defined as follows:

Definition 4.2 *A price function $p^*(t)$, investment functions $I^{i*}(t)$ and permit trade vectors $y^{i*}(t)$, for $i = 1, .., N$ form a* **market equilibrium** *if they satisfy:*
$(I^{i*}(t), y^{i*}(t))$ is an optimal solution for firm i to the problem (41) to (45), given prices $p^*(t) \geq 0$; and

$$\sum_{i=1}^{N} y^{i*}(t) = 0; \; \forall t \in [0, T] \tag{53}$$

It can be shown that such a market equilibrium exists. The proof of this is given in appendix F. Due to the assumption that firms bank if they are indifferent between banking and selling, equilibria with excess supply can now be excluded. Firms can always bank the permits that they create if they emit less than the standard.

Zero equilibrium prices can, however, not be excluded. If $p(t^*) = 0$ at some $t = t^*$, then for a firm i two possibilities exist: either $\lambda_2^i(t^*) > 0$, so that $y^i = y^i_{max}$ and firm i wants to buy a large amount of permits, or $\lambda_2^i(t^*) = 0$. The first option implies that $p(t) = 0$ cannot be a market equilibrium. The second option implies, from an analogue to proposition 4.1, that $\lambda_2^i = 0$ for all future t. This firm is then indifferent whether it receives an additional permit or not. Such an indifference implies that $p(t) = 0$ must hold for all future t. Hence, if in a market equilibrium $p(t^*) = 0$ for some $t = t^*$ then $p(t) = 0$ for all $t \in [t^*, T]$.

4 Conclusions

An optimal control model with finite time horizon was used to characterize firms' investment in emission reduction under three permit systems: one in which firms can only bank permits, one in which firms can only trade permits, and one in which they can both trade and bank. All systems involve individual requirements at the end of the program. This implies that at the end point, each firm must have invested in a larger stock of abatement capital, to have low enough emissions to satisfy its individual requirement.

Environmental policy targets with adjustment periods were defined by the government. If firms emit less than the standard at some point in time, they create an emission permit. Permit systems that allow for more or less intertemporal flexibility have been compared. The first system only allows firms to bank the permits they create for use by themselves at some later point in time. The second system only allows firms to sell created permits immediately to other firms and to buy permits if they need them. The third system combines these two and allows for both banking and trade of permits. It was shown that even a simple banking policy can result in cost savings for the firm compared to a command-and-control policy. These cost savings result, because firms get more flexibility to choose optimal investment time paths.

We described the optimal trajectories of firms' investment when the banking policy allows the firm to decide on its own intertemporal emissions distribution. Given convex investment cost functions, and compared to a standard command-and-control setting, the cost savings from banking justify a firm's earlier investment in emission reduction. A Pigouvian tax could lead to the same optimal investment paths of banking if that tax follows the 'correct' time path. That is the path defined by $\tau = \lambda_2$; the tax rate must equal the shadow value of a permit at the bank. It requires perfect information on firms' abatement costs to set the correct tax rate over time.

In a pure permit market, where any excess or deficit of permits must be instantly cleared, investment paths depend on the price path of the permits. Market equilibria with excess supply and zero prices cannot be excluded. Due to the individual end-point requirements, all firms have to invest in emission reduction at some point in time. Hence at the end of the program, no firm is allowed to buy permits to cover its emissions and there is no demand for permits any more. If both trade and banking are allowed, flexibility increases again and the permit market always clears. But equilibria with zero permit prices are still possible.

What can be said, considering these results, about the performance of the Acid Rain Program for the electricity industry in the United States? That program establishes an emission-permit market to achieve an overall emission standard and this standard is reduced in two phases. Firms are allowed to bank or trade the permits that they create by emissions lower than the standard. The program is characterized by low trade volumes and low prices of permits in the first phase (see also Burtraw, 1996). But the program has induced firms' abatement and compliance with emission standards. Since firms anticipated the standard that would hold during the first phase, they invested enough so that they would comply with that standard in time and would not need to buy permits.

The program is of course much more complex than the simple models that were analysed in this chapter. But the model in section 3.2 may be used as an approximation. May be then, low trade volumes and low permit prices in the Acid Rain Program were compensated by a successful banking policy. Since most firms had anticipated the emission standard of the program established during the first phase, they did not need to buy permits in that first phase. That implies low equilibrium prices and low volumes of trade. If prices remain low for some time, our model indicates that firms prefer to bank the permits they create, rather than sell them. There may also be other reasons why firms prefer to bank permits. For instance, they expect that they need permits in the future and that prices will rise. Or firms may not trust the functioning of the relatively new permit market and want to exclude the risk that they are not able to find a seller when they need permits. That kind of reasons are not captured by the model. Another omission in the model is that it ignores flow abatement, that is abatement through for instance input substitution, which requires recurrent expenditures rather than investment in technology. Burtraw (1996) points out that in the Acid Rain Program flow abatement through the substitution of low for high sulphur coal has played an important role. It would therefore be nice to expand the model with this additional abatement possibility.

If indeed firms banked large amounts of permits during the first phases of the program and use these permits in the second phase, cost savings have been obtained even if trade was low, since firms were allowed flexibility in the timing of their investment. So, even if for some reason firms are hesitant to sell permits, and prefer to bank them, then a permit program with trade and banking could result in cost savings compared to a command-and-control regulation, at lower information costs than an emission tax would do.

In the analysis, it is assumed that the target of environmental policy, a maximum level of emissions at some future time point, is given. That implies that we focus on the cost effectiveness of environmental policy. For an analysis of the social efficiency of environmental policy, we would also need to consider the determination of the level of this target. For a discussion of the difficulties involved in the latter and the implicit assumptions involved in the former, see Mishan (1980).

The research in this chapter is only a first attempt to characterize the investment in emission reduction by firms under various systems of permits with individual end-time requirements. That implies that a couple of questions are left for further research. For instance, we would like to analyse the effect of changes in the scrap value v_k of the capital stock at the end of the program for all three systems that were considered. It would also be interesting to extend the analysis of equilibrium price paths for the program of trade and banking, to determine the conditions for equilibria to be characterized by zero prices. This is interesting, since part of the 'failure' of the Acid Rain Program was considered to be the low price of permits in the market. Another possible extension is the incorporation of permits in the model that was analysed in chapters 2 and 3, to analyse permits in a model of strategic trade. The system that allows firms to bank permits fits best within that model, since the strategic-trade model assumes one firm in each country.

A Optimality conditions for banking

Consider the optimization problem as given by equations (8) to (14). This is an optimal control problem with two state variables, K and A, control variable, I, and a pure state constraint, equation (11). The problem is to find the optimal path of investment, which gives the optimal paths of capital and banked permits.

To obtain the optimality conditions for the optimal control problem we apply Pontryagin's maximum principle. Feichtinger and Hartl (1986) give a set of first-order conditions that can be applied for the two intervals $[0, s]$ and $[s, T]$. For each time interval, the Lagrangian is given by:

$$L = -\lambda_0 C(I) + \lambda_1(I - \delta K) + \lambda_2(\ell(t) - E(K)) + \mu A \tag{A.1}$$

Here $\lambda_0 \in R$, $\lambda_0 \geq 0$, $\lambda_1(t)$ and $\lambda_2(t)$ are two co-state variables and $\mu(t)$ is a dynamic Lagrange multiplier. Necessary conditions for an optimal solution are:

$$-\lambda_0 C'(I) + \lambda_1 \leq 0;\ I \geq 0;\ I(\lambda_1 - \lambda_0 C'(I)) = 0 \tag{A.2}$$

$$\dot{\lambda}_1 = \lambda_1(r + \delta) + \lambda_2 E'(K) \tag{A.3}$$

$$\dot{\lambda}_2 = \lambda_2 r - \mu \tag{A.4}$$

$$\mu \geq 0;\quad \mu A = 0 \tag{A.5}$$

At points τ of entry and exit of a boundary arc (which is a period of time where $A(t)$ equals zero) and at $\tau = T$, there may be a jump η in λ_2 and it must hold:

$$\eta(\tau) \geq 0\ A(\tau) \geq 0\ \eta(\tau)A(\tau) = 0\ \lambda_2(\tau^-) = \lambda_2(\tau^+) + \eta(\tau) \tag{A.6}$$

A last condition is that

$$(\lambda_0, \lambda_1(t), \lambda_2(t)) \neq (0, 0, 0)\ \forall t \in [0, T] \tag{A.7}$$

For the period $[0, s]$ the conditions above, (A.2) to (A.7), must hold and additionally

$$K(0) = k(\ell_1);\quad A(0) = 0 \tag{A.8}$$

$$K(s) = K_s;\quad A(s) = A_s \tag{A.9}$$

where this last condition defines that the optimal path must end at some given combination of $K(s)$ and $A(s)$.

For the period $[s, T]$ the conditions (A.2) to (A.7) must hold and additionally:

$$K(s) = K_s; \;\; A(s) = A_s \tag{A.10}$$

$$\lambda_1(T) \geq \lambda_0 v_k; \; E(K(T)) \leq \ell_2; \; [\lambda_1(T) - \lambda_0 v_k][E(K(T)) - \ell_2] = 0 \tag{A.11}$$

$$\lambda_2(T) \geq 0; \; A(T) \geq 0; \; \lambda_2(T)A(T) = 0 \tag{A.12}$$

Again, K_s and A_s refer to a given combination of $K(s)$ and $A(s)$.
It can be proved that $\lambda_0 \neq 0$: Assume that $\lambda_0 = 0$. Then (A.11) and (A.12) can be rewritten to:

$$\lambda_1(T) \geq 0; \; E(K(T)) \leq \ell_2; \; \lambda_1(T)[E(K(T)) - \ell_2] = 0 \tag{A.13}$$

and

$$\lambda_2(T) \geq 0; \; A(T) \geq 0; \; \lambda_2(T)A(T) = 0 \tag{A.14}$$

Four possibilities can be distinguished: First, $K(T) > k(\ell_2)$ and $A(T) > 0$. This implies that $\lambda_1(T) = \lambda_2(T) = 0$ which contradicts (A.7). Second, $K(T) = k(\ell_2)$ and $A(T) > 0$. This implies that $\lambda_1(T) \geq 0$ and $\lambda_2(T) = 0$. Since (A.2) with $\lambda_0 = 0$ implies that $\lambda_1(t) \leq 0 \; \forall t$, $\lambda_1(T) = 0$. This is again in contradiction with (A.7). Third, $K(T) = k(\ell_2)$ and $A(T) = 0$. This implies that $\lambda_1(T) \geq 0$ and $\lambda_2(T) \geq 0$. Using (A.2) it follows that $\lambda_1(T) = 0$. Hence, from (A.7), $\lambda_2(T) > 0$ must hold. Use (A.3) to see that this implies $\dot{\lambda_1}(T) < 0$. Then for small ϵ, $\lambda(T - \epsilon) > 0$. This contradicts (A.2), which states that $\lambda_1(t) \leq 0$, if $\lambda_0 = 0$ is inserted. Finally, the case $K(T) = k(\ell_2)$ and $A(T) > 0$ is not a feasible combination $K(T)$ and $A(T)$.
Summarizing, any possible combination of $A(T)$ and $K(T)$ contradicts the necessary conditions for an optimum if $\lambda_0 = 0$ is assumed. It follows that $\lambda_0 \neq 0$.
If $\lambda_0 \neq 0$ is inserted in the first-order conditions, and conditions (A.3) and (A.4) are rearranged, then the following necessary conditions that must hold for both periods can be written down:

$$\lambda_1 \leq C'(I); \; I \geq 0; \; I(\lambda_1 - C'(I)) = 0 \tag{A.15}$$

$$\frac{\dot{\lambda}_1 - \lambda_1(r + \delta)}{E'(K)} = \lambda_2 \tag{A.16}$$

$$\dot{\lambda}_2 - \lambda_2 r = \mu \tag{A.17}$$

$$\mu \geq 0; \;\; A \geq 0; \;\; \mu A = 0 \tag{A.18}$$

These are the conditions given in the main text.

B The case of command and control

For a system of command and control standards, that must be satisfied at each t, the cost minimization problem faced by the firm is:

$$\min_{I \geq 0} \left\{ \int_0^T e^{-rt} C(I) dt - v_k K(T) e^{-rT} \right\} \tag{B.1}$$

subject to the following constraints,

$$\dot{K} = I - \delta K \tag{B.2}$$

$$E(K) \leq \ell(t) \tag{B.3}$$

$$K(0) = k(\ell_1); \;\; E(K(T)) \leq \ell_2 \tag{B.4}$$

Provided that $\lambda_0 \neq 0$,[9] necessary conditions for an optimum are:

$$\lambda_1 \leq C'(I); \; I \geq 0; \; I(\lambda_1 - C'(I)) = 0 \tag{B.5}$$

$$\dot{\lambda}_1 = \lambda_1(r + \delta) + \nu E'(K) \tag{B.6}$$

$$\nu \geq 0; \;\; \nu[\ell(t) - E(K)] = 0 \tag{B.7}$$

Here ν denotes the Lagrangian multiplier of the state constraint $E(K) \leq \ell(t)$. This state constraint is of the first order. Therefore (since the problem is regular and constraint qualification is satisfied) it follows from Feichtinger and Hartl (1986) that jumps in λ_1 can be excluded. The transversality conditions are:

$$\lambda_1(T) \geq v_k; \; E(K(T)) \leq \ell_2; \; [\lambda_1(T) - v_k][E(K(T)) - \ell_2] = 0 \tag{B.8}$$

C Optimal investment paths as a function of c

To compute the optimal investment paths, I^{cac} and I^F, for $C(I) = \frac{1}{2}cI^2$, the first-order conditions (23) and (24) must be used. Since $C'(I) = cI$ and $C''(I) = c$, these can be rewritten as:

$$\dot{I} = I(r + \delta) \tag{C.1}$$

$$\dot{K} = I - \delta K \tag{C.2}$$

These equations do not contain c, so that the optimal paths I^{cac} and I^F are independent of c.

[9] The proof is trivial and omitted.

D Analysis of optimal investment paths for banking

In the main text it was derived that two possible optimal paths exist, $(p1)$ and $(p2)$. Path $(p1)$ is defined by $\mu > 0$ (and hence $A(t) = 0$) and path $(p2)$ by $\mu = 0$. Any optimal trajectory is a sequence of these two paths. In the period $[0, s]$, if the path $(p1)$ is the starting path, it implies that $K(t) = k(\ell_1)$. In the period $[s, T]$, $A(t) = 0$ along the path $(p1)$ implies that $K(t) = k(\ell_2)$.

It follows that for **the period** $[s, T]$ the first path must be $(p2)$, for values of $K(s)$ and $A(s)$ that are possibly optimal. The last path in that time interval may be either $(p1)$ or $(p2)$. If the path $(p1)$ is followed, for some interval $[t_1, T]$, then $K(t) = k(\ell_2)$, $A(t) = 0$ and $I(t) = \delta k(\ell_2)$. Before the switch point t_1, the firm follows path $(p2)$. If the path $(p2)$ is followed in some time interval, $\mu(t) = 0$ and $\lambda_2(t) = \gamma e^{rt}$ for some nonnegative value of γ.

It will be reasoned below that it is not optimal for the firm to switch from $(p1)$ to $(p2)$ in the interval $[s, T]$. It follows that an optimal trajectory for the interval $[s, T]$ consists either of the sequence $(p2)$-$(p1)$ with a switch at time t_1 or consists only of $(p2)$, which implies that $t_1 = T$.

If the firm switches from $(p2)$ to $(p1)$, this implies that $A(t_1) = 0$ and $K(t_1) = k(\ell_2)$ hold at the switching point. Furthermore, the firm has to change its investment rate, from a rate that is characterized by $\dot{\lambda}_1 = (r+\delta)\lambda_1 + \gamma e^{rt} E'(K)$ to a rate $I = \delta k(\ell_2)$ that is characterized by $\dot{\lambda}_1 = (r+\delta)\lambda_1 + \lambda_2 E'(K) = 0$. That may take place through a gradual decrease in investment. In that case, $\dot{\lambda}_1 < 0$ must hold for $t = t_1 - \epsilon$. Since λ_2 may jump down at the time $t = t_1$ when $A(t) = 0$ starts to hold, $\dot{\lambda}_1$ may jump up. That is another way to get the change in investment rates. It follows that at $t = t_1 - \epsilon$, $\dot{\lambda}_1 < 0$ must hold.

To see that it is not optimal to switch from $(p1)$ to $(p2)$,

note that if the firm would switch from $(p1)$ to $(p2)$ at some point in time t_2, this is only possible if it has already switched from $(p2)$ to $(p1)$ at a $t = t_1$ before. That implies that for $t > t_2$, the firm invests more than $I = \delta k(\ell_2)$, saves permits and its capital stock grows to values higher than $k(\ell_2)$. For $v_k = 0$, this cannot be an optimal path.

For **the period** $[0, s]$, along the optimal trajectory, the last path is not $(p1)$. If the last path were $(p1)$ then $K(s) = k(\ell_1)$ and $A(s) = 0$. This combination of $K(s)$ and $A(s)$ is clearly not optimal. The last path in $[0, s]$ is therefore $(p2)$.

It may be optimal to have $(p1)$ before $(p2)$ for some time period in $[0, s]$. It is not optimal to follow the sequence $(p2)$-$(p1)$-$(p2)$. A switch to $(p1)$ at time $t_1 \in [0, s]$ namely, requires that $K(t_1) = k(\ell_1)$. It would imply that the firm follows $(p2)$ for some period $[0, t_1]$, during which it first increases its capital stock and then decreases it. Given convex investment costs, that is not an optimal policy, because the firm gains if it keeps its stock constant at $k(\ell_1)$ instead.

Hence along an optimal trajectory, in $[0, s]$, the firm either follows $(p2)$ for all t or first starts with $(p1)$ and switches to $(p2)$ at some time t_2. Such a switch implies an increase in the rate of investment. That may take place gradually, so that λ_1 increases gradually. Or it may take place with a jump upward in $\dot{\lambda}_1$ from zero to positive values, due to a jump downward in λ_2 at the time t_2, when $A(t) = 0$ ceases to hold.

Taking the two intervals together, over **the interval** $[0, T]$, the optimal path is given as a sequence $(p1)$-$(p2)$-$(p1)$, with switches at times t_1 and t_2. It is possible that $t_2 = 0$ or $t_1 = T$, or both, in which case one of the switches is absent and the optimal path is a sequence $(p2)$-$(p1)$, $(p1)$-$(p2)$ or $(p2)$.

E First-order conditions for trade

The proofs in this and the following appendix are an adaption for our model of the proofs given in Rubin (1996). The Hamiltonian, H^i, for the optimal control problem (34) to (36) is given by:

$$H^i = -C^i(I^i) - p(t)(E(K^i) - \ell(t)) + \lambda_1^i(I^i - \delta K^i) \tag{E.1}$$

First-order conditions as stated in (37) to (39) follow from application of the maximum principle. The conditions are also sufficient, since the Hamiltonian is concave in (I^i, K^i). Furthermore, the existence theorem 2.10 in Seierstad and Sydsæter (1987, p.137) applies, so that an optimal time path $(\hat{I}^i(t))$ exists for any given nonnegative price path p(t).

A market equilibrium then exists, if there exists a $p(t) \geq 0$ such that for the trajectories $\hat{K}^i(t)$ that belong to the individually optimal investment functions $\hat{I}^i(t)$ for this price $p(t)$, for all t:

$$\sum_{i=1}^{N} E(\hat{K}^i(t)) \leq N\ell(t), \text{ with equality if } p(t) > 0. \tag{E.2}$$

This follows directly from the definition of a market equilibrium (definition 4.1), rewriting $\sum_{i=1}^{N}[E(K^{i*}(t)) - \ell(t)] \leq 0$ to $\sum_{i=1}^{N} E(\hat{K}^i(t)) \leq N\ell(t)$. It can be shown that such an equilibrium exists. To prove that, the overall cost-minimization problem is considered. This is the problem faced by a social planner and can be formalized as an optimal control problem with a first-order pure state constraint:

$$\min_{I \geq 0} \int_0^T e^{-rt} \sum_{i=1}^{N} C^i(I^i(t))dt \tag{E.3}$$

$$\text{s.t. } \dot{K} = I - \delta K \tag{E.4}$$

$$K(0) = k(\ell_1); \;\; E(K(T)) \leq \ell_2 \tag{E.5}$$

$$\sum_{i=1}^{N} E(K^i) \leq N\ell(t) \; \forall t \tag{E.6}$$

where I denotes the N-vector $(I^1, ..., I^N)$, K denotes the N-vector $(K^1, ..., K^N)$ and $E(K(T)) \leq \ell_2$ should be read as $E(K^i) \leq \ell_2 \, \forall i$. If K(t) is bounded from above, then a solution to this optimal control problem exists, which can be shown by applying theorem 5.5 in Seierstad and Sydsæter (1987, p.337). It is assumed here that v_k^i for all i. To have K(t) bounded, which is required by the theorem, it is sufficient to assume that a maximum rate of investment exists, I^i_{max}, for each i. If this rate is chosen large enough it is never a binding constraint, due to convex investment costs.

The following first-order conditions are necessary and sufficient for a social optimum rate of investment $\tilde{I}(t)$, with optimal trajectory $\tilde{K}(t)$, shadow prices $\tilde{\lambda}(t)$, and Lagrangian multiplier $\nu(t)$:

$$C^{i\prime}(I^i) \geq \tilde{\lambda}^i; \;\; I^i \geq 0; \;\; I^i[C^{i\prime}(I^i) - \tilde{\lambda}^i] = 0 \tag{E.7}$$

$$\dot{\tilde{\lambda}}^i = (r+\delta)\tilde{\lambda}^i + \nu E'(K^i) \tag{E.8}$$

$$\nu \geq 0; \;\; \sum_{i=1}^{N} E(K^i) \leq N\ell; \;\; \nu[\sum_{i=1}^{N} E(K^i) - N\ell] = 0 \tag{E.9}$$

$$\tilde{\lambda}^i(T) \geq 0; \;\; K^i(T) \geq k(\ell_2); \;\; \tilde{\lambda}^i(T)[K^i(T) - k(\ell_2)] = 0 \tag{E.10}$$

These conditions are sufficient, since the Hamiltonian is concave in (I, K). By taking $p^*(t) = \nu(t)$, it follows that a market equilibrium exists.

F First-order conditions for trade and banking

The Hamiltonian, H^i for the optimal control problem (41) to (45) is given by:

$$H^i = -C^i(I^i) - p(t)y^i + \lambda_1^i(I^i - \delta K^i) + \lambda_2^i(\ell(t) - E(K^i) + y^i), \tag{F.1}$$

while the Lagrangian is given by:

$$L^i = H^i + \mu^i A^i. \tag{F.2}$$

First-order conditions as stated in (46) to (52) follow from application of the maximum principle, for optimal control problems with mixed constraints (for example given in theorem 4.1, p.276, Seierstad and Sydsæter, 1987). The conditions are also sufficient, since the Hamiltonian, H^i is concave in (I^i, y^i, K^i, A^i). Furthermore, the existence theorem 4.2 in Seierstad and Sydsæter (1987, p.389) applies, so that an optimal time path $(\hat{I}^i(t), \hat{y}^i(t))$ exists for any given nonnegative price path p(t).

A market equilibrium then exists, if there exists a p(t) such that for all t:

$$\sum_{i=1}^{N} \hat{y}^i(t) = 0 \tag{F.3}$$

Chapter 5

Carbon and forestry: A game theory study of global climate change

This chapter analyses a transboundary pollution problem, where one player owns more possibilities to regulate its emissions than the other player. Specifically, the chapter is concerned with the role of forests in global warming. The use of fossil fuels causes greenhouse gas emissions and hence change the stock of greenhouse gases in the atmosphere. Forests are carbon pools that interact with the atmospheric carbon pool. Growing forests act as a sink for greenhouse gases since they consume carbondioxide in the growth. The use of harvested wood is a source of carbondioxide when the wood ultimately decomposes or is burnt.

We introduce an asymmetric two player game, where one player is supposed to have a large existing forest, while the other player has no forest. We set a first step to analyse the possible conflicts and the potential cooperative gains between the two players. We analyse the social optimal rates of fossil fuel use and forest harvest and the Nash equilibrium outcome, where each country only minds its own net welfare. Equilibria when both countries ignore carbon sequestration by forests and carbon emission by harvested wood are compared to the equilibria when these carbon flows are included in their considerations.

1 Introduction

The use of fossil fuels in energy production emits carbon dioxide (CO_2) into the atmosphere. Carbon dioxide is known to be one of the major greenhouse gases and to contribute to the worldwide pollution problem, referred to as global climate change. Forest ecosystems are a large stock of carbon. They contain about 45% of the total terrestrial carbon (Kellomäki and Karjalainen, 1996). The role of forests in the greenhouse gas dynamics is twofold. Forest growth consumes CO_2 from the atmosphere. But the use of harvested wood will ultimately cause greenhouse gas emissions.

The role of forests in the carbon cycle has been analysed by a number of papers (see, for example, the contributions in Apps and Price (1996); Wisniewski and Lugo (1992) and the overview by Brown (IPCC-II, chapter 24, 1996)). From the various contributions, it appears that much uncertainty still exists. Points of discussion are, for instance, the role of soil pools of carbon and the fate of harvested wood. The soil pool may be a source if decomposition and emission into the atmosphere is large or it may be a sink if organic material is transferred into long lasting soil pools. It also makes a difference to the net carbon sequestration[1] result, whether harvested wood is turned into newspapers, into construction wood or serves as a substitute to fossil fuels. See for example Matthews (1996), who gives an overview of research on carbon budgets and explains some of the different results by different assumptions regarding the destination of harvested wood.

The references above usually concentrate on physical carbon budgets. Other contributions discuss the economic consequences when the role of forests in the carbon cycle is explicitly taken into account in decisions concerning the management of forests. A good overview is provided by Sedjo et al. (1995). The approach in the literature discussed there usually is to incorporate climate considerations into forestry models. Other contributions are Tahvonen (1995) and Backlund et al. (1996), who incorporate forest dynamics into a growth model with climate change.

In 1990, the cooperating electricity producers in the Netherlands (SEP) set up a foundation called Face, 'Forests absorbing carbon dioxide emission', which finances reforestation projects in various countries.[2] The objective of the electricity companies is to use the carbon that these forests absorb in their growth to fulfil part of their carbondioxide emission reduction obligations. The producers hence claim that not their gross emissions should be measured, but emissions net of what they cover by reforestation.

Countries that possess a large area of forests within their borders may make similar claims. They may argue that the net emissions from their country should be considered and not the gross emissions. When that is done and absorption by forests is accounted for, a country like Finland might even be characterized by negative emissions, since accumulation of carbon exceeds the emissions from fossil fuels by 25-50% (Kauppi and Tomppo, 1993). However, Karjalainen and Kellomäki, 1993 (cited in IPCC (1996)) give considerably smaller estimates. According to their numbers, only 28% of Finland's emissions is covered by sequestration. Still, it is possible that these countries claim that as a result of this, they can reduce less fossil fuel, contributing to a worldwide reduction in carbon emissions through the sequestration from their forests.

This chapter was motivated by the claims mentioned above. A simple differential game model of two countries is analysed, where one country has large forests and the other none. Within this model we first check how the cooperative solution changes when forests and their absorption of greenhouse gases are included in the optimization. That is, we compare cooperative solutions

[1] The absorption of carbon from the atmosphere by growing forests that transform it into biomass is also called carbon sequestration.

[2] Projects have been started in Ecuador, Malaysia, Czech republic and Uganda (van der Burg, 1994).

for the gross emissions case with the net emissions case. Here 'gross emissions' refer to the solution when carbon dynamics of forests are ignored, while 'net emissions' refer to the solution where carbon sequestration and the carbon emissions from forest harvest are included. Second, it is analysed whether in a noncooperative setting, where each country determines its own emissions and abatement efforts, net emissions make any difference. Since it analyses two countries and their interactions, the chapter differs from contributions such as Tahvonen (1995) and Backlund et al. (1996).

How large the potential of carbon abatement from forests is and especially how long it lasts is subject to some discussion. A 'healthy' forest may always act as a (small) net sink of carbon, if the soil is a lasting sink of carbon (for this socalled 'carbon-pump' hypothesis, see Kauppi (1996)). Other authors, though, give little attention to this role of the soil and stress that forests can only temporarily act as a sink for carbon (C) (see, for instance, Cline, 1992). The IPCC-report (1996) mentions that 'In addition to managing forest vegetation to conserve or sequester carbon, there also is an opportunity to manage forest soils for the same purposes' and refers to Johnson (1992), Lugo and Brown (1993) and Dixon et al. (1994). But the report also stresses that 'Sequestering C by storage management produces only a finite C sequestration potential in vegetation and soils, beyond which little additional C can be accumulated.' (from IPCC, 1996). The gain from enlargement of this temporary sink may be that it shifts the time path of accumulating carbon to the future. In that way, enhanced sequestration by forests may 'buy time' to adjust to alternative ways to reduce carbon emissions, for instance by improved energy saving technologies.

In the model below, we include a parameter β to analyse the effect of these two different views. If β is set equal to one, this reflects the view that forests are only a temporary sink of carbon. If β is set larger than one, this implies that forests are assumed to be a permanent net sink of carbon. The larger β, the more important the role of carbon sequestration is. Given the references cited above, it seems most reasonable that β equals one or is very close to it.

It should be noted that the growth of forests adds to carbon abatement, not the volume of forests as such. The harvested forest also adds to atmospheric carbon eventually. Forests that are managed for production, are in a stationary state if all age classes until the age of harvest are equally present. Then a constant harvest of the oldest trees is possible provided that the harvested trees are replaced with young trees. If carbon sequestration is taken into account, the forest management will adjust such that the forest contains more carbon. This implies a longer rotation period (see, for instance, Sedjo et al., 1995). The average volume of the trees in the forest increases. It is not clear, however, what happens to average growth. This may increase or decrease. A trade off must be made between the carbon released at harvest and the carbon sequestered during growth.

The model in this chapter uses a simplified formalization of forest growth and the relations between volume, growth and harvest rates. The simplifications that were made are explained when the model is introduced in section 2. It is found that the global optimum allows for larger rates of fossil fuel use compared to the gross emissions case, where carbon dynamics in forests are neglected. Furthermore, for large initial values of the forest, in the global optimum forests are adjusted such that forest volumes are higher and harvests are lower. That is, more carbon is stored in the forest and less is released from harvest, as well as sequestered through growth. The chapter is organized as follows. Section 2 introduces a game-theoretic model of the problem. In section 3 the conditions for the cooperative solution are discussed for the case of net emissions and for the case of gross emissions and these are compared. Section 4 considers noncooperative equilibria. Section 5 concludes.

2 The game of forest and coal

2.1 The net emissions case

Let us first have a look at the dynamics of the different components of the resource system. The dynamics of forestry growth are represented by the function:

$$dx/dt = F(x) - h \tag{1}$$

where $x(t)$ is the stock of forests (volume of wood in the forest) at time t, F is the growth rate of this stock, and h is the human harvest of forest products. The variables x, h and $F(x)$ are measured in terms of carbon content, so that $F(x)$ denotes the increase in the carbon stored in the forest, x denotes the stock of carbon stored in the forest pool and h denotes the carbon contained in harvested products. It is assumed that $0 \leq h(t) \leq h^{max}$ for each t, where h^{max} is some upper boundary on the harvest rate. The growth function, $F(x)$, is concave in x, with $F(x) \geq 0$ for $x \in [0, x^m]$, $F'(x) > 0$ for $x < x^h$ and $F'(x) < 0$ for $x > x^h$, where $x^h \in (0, x^m)$ is the volume where growth is at its maximum, $F'(x^h) = 0$.
This way to model forest growth abstracts from age classes. The changes in the volume of carbon stored in forest stock are modelled as a function of the present volume stored. This is a rough simplification, since the distribution of trees of different ages, which is relevant for their growth rate is not specified and rotation periods are abstracted away. Forestry literature usually employs models where the rotation period is explicitly included (see Neher, 1990). The link between the two types of model is given by the observation that an increase in the total stock of carbon stored in a forest requires a longer rotation period and a larger number of age classes. The underlying assumption is that a given and fixed area of forest land is considered.[3] This land is equally divided over plots of different age classes. The oldest plot is harvested. It is assumed that up to some maximum rotation period the carbon content of the stationary harvest[4] increases with the rotation period, at a rate larger than the increase in carbon stored by the total forested area (the total stock), while after that maximum, growth increases less than the increase in the total stock. If then the total stock is the parameter at the horizontal

[3]See Wirl (1997) for an analysis of an optimal control model of forest growth that allows for changes in the area of forested land, but ignores the dynamics of carbon in the atmosphere.

[4]The stationary harvest is the harvest of only the oldest plot, which in case of replantation leaves the forest in a stationary state. Note that an increase in the rotation period implies more and therefore smaller plots. Hence, if the rotation period increases, a smaller area of larger trees is harvested.

axis and harvest is the parameter at the vertical axis, a hump-shaped plot is obtained. In the model, this plot is approximated through $F(x)$. For the purposes in this chapter, this is a useful simplification.

The observation in the introduction concerning countries with large existing forests that claim that net emissions should count, would then point to a situation with relatively large initial stocks. The other observation concerning (re-)forestation by countries with no initial forest would point to a situation with small initial stocks.

The evolution over time of the stock of greenhouse gases in the atmosphere, Q, can be described as

$$dQ/dt = -\alpha Q - \beta F(x) + h + e, \tag{2}$$

where e, $0 \leq e$, denotes the emissions of CO_2 from the use of fossil fuels. The term $-\alpha Q$ denotes the assimilation of greenhouse gases in other parts of the ecosystem, mainly oceans. The term $-\beta F(x)$ denotes the amount of greenhouse gases assimilated by forest growth. If $\beta > 1$ then forests can be said to act as a net sink, sequestering more carbon than is released if harvested at a sustainable rate, that is, at a rate $h = F(x)$. Together, equations (1) and (2) define a dynamic system that roughly describes part of the carbon cycle. Carbon is added to this system by burning of fossil fuels, while it disappears from it by the ocean sink.

Let us next study the economic behaviour of the countries. Both countries must take the dynamics of greenhouse gases into account in their decision making, thus turning the model into a dynamic game. We assume that the countries do not like an increased stock of greenhouse gases, because they fear that global climate change might harm ecological and economic systems. The country with forest is referred to as the forest country, while the other country will be called the coal country in the sequel.

Consider first the forest country. For a given stock of greenhouse gases it chooses its fossil fuel use, e^f, and forest harvest, h, to maximize, $U^f(e^f) + R(h)$, its utility from energy use and forestry, which is assumed to be separable in the two choice variables. The country must thereby take into account the dynamics of forestry growth (1) and greenhouse gases (2). Instantaneous welfare of this country is given by $U^f + R - D^f(Q)$, where $D^f(Q)$ denotes damage due to the stock of greenhouse gases (CO_2) in the atmosphere. Damage increases at an increasing rate, $D^{f\prime}(Q) > 0$, and $D^{f\prime\prime}(Q) \geq 0$. Harvest of forests gives positive utility, since the products are used for the production of consumption goods: $R(h) \geq 0$, $0 \leq h \leq h^{max}$. Marginal utility is assumed to be positive for low rates of harvest, $R'(0) > 0$, decreasing in h, $R''(h) < 0$, and negative for large rates of harvest, $R'(h^{max}) < 0$. Furthermore it is assumed that U^f is concave and increasing in e^f, $U^{f\prime}(e^f) > 0$, $U^{f\prime\prime}(e^f) < 0$. The decision problem for the forest country can now be presented as

$$\max_{0 \leq h \leq h^{max}, e^f \geq 0} J^f(e^c, e^f, h) = \int_0^\infty e^{-rt}[U^f(e^f) + R(h) - D^f(Q)]dt \tag{3}$$

$$\text{s.t. } \dot{x} = F(x) - h \tag{4}$$

$$\dot{Q} = -\alpha Q - \beta F(x) + h + e^c + e^f \tag{5}$$

$$x(t) \geq 0;\;\; x(0) = x_0 > 0; \tag{6}$$

$$Q(t) \geq 0;\;\; Q(0) = Q_0 \geq 0; \tag{7}$$

Here e^c denotes energy use by the coal country.

Consider next the coal country. This country only considers the level of the use of fossil fuels that will maximize its welfare. For simplicity, we abstract from the exhaustibility of fossil fuels and assume that the coal country is able to buy unlimited amounts of them. The welfare function of this country is given as $U^c(e^c) - D^c(Q)$. For U^c and D^c assumptions analogous to those on U^f and D^f are made, that is, $U^{c\prime}(e^c) > 0, U^{c\prime\prime}(e^c) < 0$, $D^{c\prime}(Q) > 0$ and $D^{c\prime\prime}(Q) \geq 0$. Summarizing, the coal country solves the following infinite time optimization problem:

$$\max_{e^c \geq 0} J^c(e^c, e^f, h) = \int_0^{\infty} e^{-rt}[U^c(e^c) - D^c(Q)]\, dt \tag{8}$$

subject to (4) to (7). Note that although the growth of forests affects welfare of the coal country, it cannot directly influence this, because the forests are not under its jurisdiction.

2.2 The gross emissions case

For comparison, consider a model where the role of forests in carbon dynamics is neglected. Thus instead of (2), the following equation is used

$$\dot{Q} = -\alpha Q + e^f + e^c \tag{9}$$

When equation (9) replaces (2) the countries' decision problems, (3) to (7) or (8) break down into two separate problems. These concern the optimal use of two renewable resources, forest and an atmosphere of good quality. The forest country hence has to optimize its use of forest, by maximizing $\int_0^T e^{-rt}[R(h)]dt$ subject to (4) and (6)[5] and to optimize its use of fossil fuels, by maximizing $\int_0^T e^{-rt}[U^f(e^f) - D^f(Q)]dt$ subject to (9) and (7).[6] For the coal county, the first decision problem, concerning the optimal use of forest is trivial, since it cannot influence the use of forest. It must optimize on its use of fossil fuels by maximizing (8) subject to (9).

2.3 Example

The analysis will be illustrated by an example, that uses explicit functional forms. These functional forms are chosen extremely simple, to focus on the interpretation of the results. The

[5] This is the well known problem of the optimal harvest of a renewable resource. See for instance Feichtinger and Hartl, 1986, p.499.

[6] This simple model contains the greenhouse problem in a very abstract form. See, for instance, van der Ploeg and de Zeeuw (1992).

following functional forms are assumed for the cost functions of the players and the dynamics of the systems Let $i = f, c$ denote the country with and without forest respectively. Then:

$$R(h) = h(\phi - \frac{1}{2}\psi h) \tag{10}$$

$$U^i(e^i) = e^i(\chi - \frac{1}{2}\gamma e^i) \text{ for } i = f, c \text{ }^7 \tag{11}$$

$$D^i(Q) = dQ \quad \text{for } i = f, c \tag{12}$$

$$F(x) = b - Bx, \tag{13}$$

where ϕ, ψ, χ^i, γ^i, d, b, and B are positive constants. The last equation implies that the focus in the example is on the righthand side of the growth function $F(x)$ as it was defined in (1). That is, it is assumed that the initial stock of forest is large ($x(0) > x^h$), or in other words, that initially rotation periods are long. If the stock increases, growth decreases, because when rotation periods become longer, the additional volume of harvested trees cannot make up for the decrease in the harvested plot. With $b = 0$ and $B < 0$, the other case with $F'(x) > 0$ could be considered.

3 The cooperative solution

This section considers rates of fossil fuel use and forest harvest that are optimal from the point of view of a social planner that optimizes total welfare. The resulting maximal total welfare is what the two countries can divide between them, if they cooperate and side payments are possible. If side payments are not possible, cooperation between the two countries results in one of a set of Pareto optimal points. These are found from the maximization of one country's welfare, keeping the other country's welfare at some minimum level. In general, not all of these points have maximal total welfare. Whether side payments are possible or not, which division results depends on the relative bargaining power of each country. For example, the Nash bargaining solution relates bargaining power to the maximum welfare that each country can obtain in the Nash noncooperative equilibrium.

3.1 Cooperative solution for the net emissions case

For the case where net emissions count, the social planner's optimization problem is:

$$\max_{e^f, e^c, h} \int_0^\infty e^{-rt}[U^f(e^f) + R(h) - D^f(Q) + U^c(e^c) - D^c(Q)]dt \tag{14}$$

[7] This utility function does not satisfy $U'(.) > 0$ for all e, but is nevertheless chosen here for simplicity. The additional requirement that $e^i \leq \frac{\chi}{\gamma}$ must be added.

subject to (4) to (7) and

$$e^f(t) \geq 0; \ \ e^c(t) \geq 0; \ \ 0 \leq h(t) \leq h^{max} \tag{15}$$

For analytical simplicity, assume in the sequel that $U^f(.) = U^c(.) = U(.)$ and $D^f(.) = D^c(.) = D(.)$ and assume interior solutions (that is, neglect the nonnegativity constraints on the state variables for the moment). Hence, the only asymmetry between the countries is their possession of forests. Let λ and μ denote the shadow variables corresponding to the pollution stock and the renewable resources respectively. Necessary conditions for an optimal interior solution to this optimal control problem are:

$$U'(e^f) + \lambda = U'(e^c) + \lambda = 0 \tag{16}$$

$$R'(h) + \lambda - \mu = 0 \tag{17}$$

$$d\lambda/dt = (r + \alpha)\lambda + 2D'(Q) \tag{18}$$

$$d\mu/dt = (r - F'(x))\mu + \beta\lambda F'(x) \tag{19}$$

When moreover the condition

$$\mu - \lambda\beta \geq 0 \tag{20}$$

is satisfied, *H* is concave in (Q, x, e^f, e^c, h) and the Mangasarian sufficiency theorem for infinite horizon problems (Seierstad and Sydsæter, 1987, p.234) can be applied. It follows that sufficient conditions for an optimal solution are given by (16) to (19) together with (4), (5) and

$$\lim_{t\to\infty} e^{-rt}\lambda(t) = 0 \tag{21}$$

and

$$\lim_{t\to\infty} e^{-rt}\mu(t) = 0 \tag{22}$$

provided that $Q(t)$ and $x(t)$ are bounded.

From condition (16), it follows immediately that it is optimal to choose $e^f = e^c$ in the cooperative solution. This is explained by the symmetry assumptions and the separability of welfare in fossil fuel use and forest harvest. Denote the common optimal rate of energy use by $\hat{e}(\lambda)$.

For the moment, assume that an optimal solution exists which converges to a stationary point $(\bar{e^f}, \bar{e^h}, \bar{h})$ with steady-state values $(\bar{Q}, \bar{x})$. The stability of the steady state(s) will be analysed below on page 109. Given the assumption of convergence, condition (18) can be integrated forward and inserted in (16) to give the net present value expression:

$$U'(e^i(t)) = \int_t^{\infty} e^{-(r+\alpha)(s-t)}[2D'(Q(s))]ds \tag{23}$$

Or in words, along a cooperative solution, marginal utility from fossil fuel use should equal discounted marginal damage in both countries due to the addition made by fossil fuel use at time t to future greenhouse gas stocks. This is the dynamic version of the usual condition on marginal benefits and marginal costs.

Let h^0 denote the rate of harvest such that $R'(h^0) = 0$. Then from the condition (17) and $R''(h) < 0$ it follows that $\hat{h}(\lambda, \mu)$, the optimal cooperative harvest rate, is smaller than h^0 whenever $\mu > \lambda$.

Provided the optimal path converges to a steady state, forward integration of (18) and (19) gives the following net present value expression for the optimal harvest rate:

$$R'(h(t)) = \int_t^{\infty} e^{-(r+\alpha)(s-t)}[2D'(Q(s))]ds - \int_t^{\infty} e^{-r(s-t)+\int_t^s F'(x(u))du}[\lambda\beta F'(x(s))]ds \tag{24}$$

Marginal revenues from harvest of wood must equal marginal damage due to the addition made to future greenhouse gas stocks, corrected for changes in abatement due to the stock adjustment that takes place in forests and that may stimulate growth and hence carbon sequestration (when $F'(x(s)) < 0$) or reduce growth and sequestration (when $F'(x(s)) > 0$). The changes in abatement are valued at the shadow price of carbon in the atmosphere, $\lambda(s)$. The proper discount rate in the correction term includes $\int_t^s F'(x(u))du$. This is because forests are a renewable resource and grow at rate F(x) if left to themselves. Hence any marginal addition Δ to x at time t, contributes to growth and will have grown to $\Delta e^{\int_t^s F'(x(u))du}$ at time s.

Now sufficient conditions for stability and optimality of steady states will be given so that the forward integrations that were made to arrive at net present value expressions are justified. A steady state, $(\bar{Q}, \bar{x}, \bar{\lambda}, \bar{\mu})$, is defined by the set of equations:

$$(r + \alpha)\lambda = -2D'(Q) \tag{25}$$

$$(r - F'(x))\mu = -\lambda\beta F'(x) \tag{26}$$

$$\alpha Q = -\beta F(x) + \hat{h}(\lambda, \mu) + 2\hat{e}(\lambda) \tag{27}$$

$$F(x) = \hat{h}(\lambda, \mu) \tag{28}$$

which follows from (4), (5) and (16) to (19), when the time derivatives are set equal to zero and $\hat{e}$ and $\hat{h}$ are inserted. Multiple steady states are possible, since (28) usually has two solutions of x, for a given $\hat{h}$, due to the concavity of the growth function $F(x)$. In appendix A, the following sufficient conditions for a steady state to be saddle-point stable are derived: For $\beta = 1$, the steady state is saddle-point stable if $F'(\bar{x}) \leq 0$, or if $F'(\bar{x}) \geq r$.

For $\beta > 1$, sufficient conditions for stability are:

$$\bar{\mu} - \bar{\lambda}\beta \geq 0 \tag{29}$$

$$F'(\bar{x})(r - F'(\bar{x})) < 0 \tag{30}$$

and

$$F'(\bar{x})(1-\beta)(r-(1-\beta)F'(\bar{x})) \leq 0 \tag{31}$$

Condition (29) also guarantees that the Hamiltonian is concave, so that any steady state that satisfies (29) is indeed optimal. Both conditions (30) and (31) are satisfied if either

$$F'(\bar{x}) \leq \frac{-r}{\beta - 1}, \tag{32}$$

or

$$F'(\bar{x}) \geq r \tag{33}$$

These conditions are sufficient, but not necessary for stability, which can be seen in section 3.3, when they are applied to the example. The conditions (32) and (33) imply that stability cannot be guaranteed for intermediate values of $F'(x)$ (with $\frac{-r}{\beta-1} < F'(x) < r$).

3.2 Comparison with gross emissions case

In the model which we call the gross emissions case, forests are set to their optimal level independent of carbon dynamics. Thus only $R(h)$ counts as a source of welfare from forestry. In the gross emissions case, the first-order conditions (16) and (18) remain valid, while the conditions (17) and (19) change to

$$R'(h) = \mu \tag{34}$$

and

$$\dot{\mu} = (r - F'(x))\mu \tag{35}$$

Steady states $(\bar{x}, \bar{Q}, \bar{\mu}, \bar{\lambda})$ are given by the system of equations:

$$(r+\alpha)\lambda = -2D'(Q) \tag{36}$$

$$(r - F'(x))\mu = 0 \tag{37}$$

$$\alpha Q = 2\hat{e}(\lambda) \tag{38}$$

$$F(x) = \hat{h}(\mu) \tag{39}$$

Note that (37) allows for two solutions. Either $F'(x) = r$ or $\mu = 0$ ensures that this condition holds. As a consequence, multiple steady-state stocks of forest exist. A stability analysis of the steady states for this model is given in appendix A.

Denote the results in the model that considers net carbon emissions by superscript F and those in the model that considers gross carbon emissions by superscript NF. Rewrite the first-order conditions of the model for the gross emissions case to find that the optimal rate of growth in forest harvest, when forests' contribution to carbon accumulation is ignored, must satisfy:

$$(\frac{\dot{h}}{h})^{NF} = \frac{R'(h)}{hR''(h)}(r - F'(x)) \tag{40}$$

This is the standard rule for optimal extraction of a renewable resource with zero extraction costs (See e.g. Feichtinger and Hartl, 1986, p449). For the model that includes forests, an analogous condition is:

$$(\frac{\dot{h}}{h})^{F} = \frac{R'(h)}{hR''(h)}(r - F'(x)) + \frac{-(\beta - 1)F'(x)U'(e^i)}{hR''(h)} + \frac{\alpha U'(e^i) - 2D'(Q)}{hR''(h)} \tag{41}$$

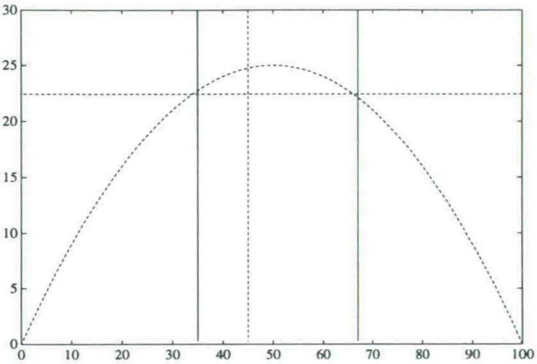
The standard rule must be corrected for the direct contribution of harvest to carbon dioxide accumulation, $\frac{\alpha U'(e^i)-2D'(Q)}{hR''(h)}$, and for the stock effects (harvest changes the growth rate of forests, which in turn changes the rate of abatement of carbon), $\frac{-(\beta-1)F'(x)U'(e^i)}{hR''(h)}$. The marginal utility of energy use appears in this latter term, because it is the opportunity costs of changes in the rate of carbon sequestration. This term is zero if $\beta = 1$, in which case the sequestration effect cancels out in the long run and therefore does not affect the growth rate of h. It does, however, affect the optimal path of harvest as can be seen from the net present value expression (24). For $\beta > 1$ and $F'(x) < 0$, the term is negative, which implies that the growth rate of harvest should be adjusted downward. It should be adjusted more, the larger $U'(e)$, β or $|F'(x)|$. If the opportunity costs of no sequestration in the form of lower energy use are high or carbon sequestration is easy, it is better to have less growth in harvest rates. That is, for a harvest rate that is increasing over time a flatter shape of the curve $h^F(t)$ is better. This flatter curve then also lies at a higher level as can be seen from expression (24).
A comparison of the optimal steady-state use of fossil fuels in the two models leads to the following result:

Proposition 5.1 $\bar{e}^F \geq \bar{e}^{NF}$.

To prove this, use the definition of $\hat{e}(\lambda)$ by (16) which must hold in both cases. The result then follows from a comparison of (25) and (27) with (36) and (38) (a formal proof is provided in appendix B).
When $\beta = 1$, the steady-state conditions for $\bar{Q}$ and $\bar{e}$ are the same in both models, so that then the inequality becomes an equality. Indeed, when $\beta = 1$, the inclusion of carbon dynamics in decisions concerning forests has only a temporary effect on carbon dioxide accumulation and energy use. It is less insightful to compare the steady-state levels of carbon dioxide in the atmosphere, since the two models reflect two different ideas about carbon dynamics.
Now insert $\bar{\lambda}$ and $\bar{\mu}$ in (17) respectively (34) to find:

$$R'(\bar{h}^F) - \frac{2D'(\bar{Q})}{r + \alpha}[1 + \frac{\beta F'(\bar{x})}{r - F'(\bar{x})}] = 0 \tag{42}$$

Figure 1: Comparison of steady-state forest-stocks, for net emissions and gross emissions.



$F(x) \uparrow$ $x \rightarrow$ x^1 x^r x^2

and

$$R'(\bar{h}^{NF}) = 0 \text{ or } R'(\bar{h}^{NF}) = R'(h^r) \tag{43}$$

where h^r is defined by $h^r = F(x^r)$ and x^r defined by $F'(x^r) = r$. Remember that by its definition, $R'(h^0) = 0$. It follows from (43) that in any stationary point of the model without carbon sequestration, either $\bar{h} = h^0$, or $\bar{h} = h^r$. If $h^r > h^0$, then $R'(h^r) < 0$ and $\bar{x}^{NF} = x^r$ is not a stable steady state (see appendix A). Furthermore, it follows from $h^r > h^0$ that the two steady states, denoted by x^1, respectively x^2, that satisfy $F(x^i) = h^0$ are both saddle-point stable (see appendix A). For $h^r < h^0$, similar results can be derived, which are ignored here. Figure 1 shows the relative position of the three possible steady states.
A comparison of $\bar{h}^F$ and $\bar{h}^{NF}$, as well as $\bar{x}^F$ and $\bar{x}^{NF}$ is given in the following two propositions. The first proposition considers the important case where the sink capacity of forests is only temporary, so that $\beta = 1$:

Proposition 5.2 *For $h^r > h^0$, a steady state to the forest model exists that is characterized by: $x^2 < \bar{x}^F$ and $\bar{h}^F < h^0$. This steady state is locally saddle-point stable.*
Furthermore, in this case, the forest model has at most two other steady states that are characterized by $x^1 < \bar{x}^F < x^r$ and $\bar{h}^F > h^0$.

A proof is given in appendix B. It may be concluded that when forests are only a temporary sink of carbon, if initial forests are large enough, it is optimal to converge to the steady state characterized by $\bar{x}^F > x^2 = \bar{x}^{NF}$. That implies that $\bar{x}^F$ has lower growth and steady-state harvest rates than $\bar{x}^{NF}$. The reason for that is that the direct addition to damage from harvest dominates the contribution to abatement from growth of forests. Then it is optimal to adjust forest such that harvest rates and growth are lower than would be the case when

forest-atmosphere interactions were neglected. As a result, the stock of forest in the steady state is larger when sequestration is taken into account. For small initial forests, it is optimal to converge to the steady state characterized by $\bar{x}^F > x^1 = \bar{x}^{NF}$. Then the direct addition to damage from harvest is dominated by the contribution to abatement from growth. It is then optimal to have larger harvest rates than would be the case when sequestration was neglected. Growth rates are also larger and the stock of forests in the steady state is larger than when sequestration is neglected.
A second proposition considers the case of $\beta > 1$.

Proposition 5.3 *For $h^r > h^0$, the forest model has at least one steady state larger than x^r. This steady state is characterized by the equations (25) to (28) and $F'(\bar{x}) < 0$. When in such a steady state $\beta < 1 - \frac{r}{F'(\bar{x})}$, it is characterized by $\bar{x}^F > x^2$ and $\bar{h}^F < h^0$. and it is locally saddle-point stable. When in such a steady state $\beta > 1 - \frac{r}{F'(\bar{x})}$, it is characterized by $\bar{x}^F < x^2$ and $\bar{h}^F > h^0$. For the special case $\beta = 1 - \frac{r}{F'(x^2)}$, the steady state is given by: $\bar{x}^F = x^2$ and $\bar{h}^F = h^0$.*
There may exist at most two other steady states, given by: $x^1 < \bar{x}^F < x^r$ and $h^0 < \bar{h}^F < h^r$.

A proof is given in appendix B. It follows that the optimal steady-state forest in the net emissions case, $\bar{x}^F$, is larger than the optimal steady-state forest in the gross emissions case, $\bar{x}^{NF}$, when the sink function of forests, β, is small and the initial forest is large. For a steady state to the right of x^r, it implies that harvest rates must be lower in the net emissions case. When the sink function is large enough, these results are reversed. The abatement effect from more forest growth then dominates the direct contribution to damage from harvested forests. As a result, for large initial forests, the steady-state forest under net emissions is smaller than under gross emissions so that it grows faster and abates more.
An interesting question is whether forests ultimately grow faster or slower when carbon-sequestration effects are accounted for and harvest decisions adjusted. For an answer, first compare the optimal growth paths of harvest and forest stocks for the net and gross emissions case. Even in this relatively simple model, general results are not easy to obtain, due to the existence of multiple steady states. But it is possible to compare optimal paths for the special case where $\beta = 1$, so that forests are only a temporary sink of carbon. Using the results from propositions 5.1 and 5.2 and the optimality conditions (16) to (19), (34) and (35) it can be found that, for given initial conditions:

Proposition 5.4 *Let $e^{min} < e^F(t)$ and $Q^{min} < Q^F(t)$, where e^{min} and Q^{min} satisfy $(r + \alpha)U'(e^{min}) < 2D'(Q^{min})$. Assume that $x(0) > x^r$. Then, for $\beta = 1$, for all $t > 0$, $h^F(t) < h^{NF}(t)$ and $x^F(t) > x^{NF}(t)$.*

A proof is given in appendix B. The proposition says that the harvest rates of forest are lower in the net emissions case than in the gross emissions case, provided that the condition mentioned is met. Since α is probably a small number, that condition is expected to be satisfied. For a given initial stock, $x^r < x(0)$, this implies that $x^F(t) > x^{NF}(t)$, since harvest rates are lower, so that stocks can grow at a higher rate (or decrease at a lower rate).

To summarize, a social planner that chooses to count net rather than gross emissions, will choose a lower harvest time path. The explicit recognition of an additional abatement opportunity in the form of carbon sequestration by forests leads to a new balance between the damage from greenhouse gases and the utility derived from fossil fuel use and forest harvest. The next section again compares rates of fossil fuel use and forest harvest, for the two alternative models, but now for the case of a noncooperative equilibrium between the two countries. But first, the results and propositions above are illustrated in the example.

3.3 Example

For our example with linear damage, the first-order necessary conditions for a cooperative optimum for net emissions can be found in appendix C. The steady-state values of the variables are also given there. In this steady state, the controls take the values:

$$\hat{h}^F = \frac{\phi}{\psi} + \frac{2d(B(\beta-1)-r)}{\psi(r+\alpha)(r+B)} \tag{44}$$

and

$$\hat{e}^{iF} = \frac{\chi}{\gamma} - \frac{2d}{\gamma(r+\alpha)}. \tag{45}$$

One needs to check the nonnegativity of $\bar{Q}$,$\bar{x}$,$\hat{h}$ and $\hat{e}^i$, to ensure that this steady state makes sense economically. Only a restricted range of parameter values results in such an interior solution (See appendix D).

Consider the local stability of this steady state. Condition (32) is satisfied for $B \geq \frac{r}{\beta-1}$. For large enough B, it may be checked that in the steady state the three sufficient conditions (29) to (30) are all met and the steady state is stable. However, the example is so simple, that it is also possible to directly determine the eigenvalues of the Jacobian, J. The eigenvalues of J are $-\alpha$, $-B$, $r+B$, and $r+\alpha$ (See appendix C). Since there are two positive and two negative eigenvalues, the steady state is saddle-point stable, whatever the value of B. The sufficient conditions for stability (29) to (31) are too restrictive for this simple example.

An equilibrium convergent path is given as

$$Q(t) = c_1 e^{-\alpha t} + c_2 e^{-Bt} + \bar{Q} \tag{46}$$

$$x(t) = (x_0 - \bar{x})e^{-Bt} + \bar{x} \tag{47}$$

$$\lambda(t) = \bar{\lambda} \tag{48}$$

$$\mu(t) = \bar{\mu} \tag{49}$$

with $c_1 = Q_0 - \bar{Q} - c_2$ and $c_2 = \frac{\beta B(x_0 - \bar{x})}{\alpha - B}$. Marginal damage determines a constant shadow value of pollution, affected only by α and the discount rate. This shadow value determines

the optimal rates of fossil fuel use in the two countries, $\hat{e}^i$, that are hence also constant over time. Together with the constant shadow value of forest stock, the shadow value of pollution determines the optimal harvest rate of forest, $\hat{h}$, that is then also constant over time. The stock of forest monotonically adjusts until it equals its steady-state value. Finally, the stock of greenhouse gases adjusts to its steady-state value along an adjustment path which need not be monotone.
For the **gross emissions case**, the steady-state values of the controls are given by:

$$\hat{h}^{NF} = \frac{\phi}{\psi} \tag{50}$$

and

$$\hat{e}^{i,NF} = \frac{\chi}{\gamma} - \frac{2d}{\gamma(r+\alpha)}. \tag{51}$$

The values of the state and shadow variables in the steady state can be found in appendix C. Again, nonnegativity requirements restrict the range of parameters for which these are valid equations (see appendix D). The steady states are stable, as is easily seen from the eigenvalues of the two separate dynamic systems. Convergent equilibrium paths then take the form of:

$$Q^{NF}(t) = (Q_0 - \bar{Q}^{NF})e^{-\alpha t} + \bar{Q}^{NF} \tag{52}$$

$$x^{NF}(t) = (x_0 - \bar{x}^{NF})e^{-Bt} + \bar{x}^{NF} \tag{53}$$

$$\lambda^{NF}(t) = \bar{\lambda}^{NF} \tag{54}$$

$$\mu^{NF}(t) = 0 \tag{55}$$

For a comparison of steady states in the two alternative models, apply proposition 5.1 to 5.3 to the example. It can be concluded that:

1 $\bar{e}^F \geq \bar{e}^{NF}$

2 For $\beta = 1$, since $F'(x) = -B < r$, $\bar{x}^F \geq \bar{x}^{NF}$ and $\bar{h}^F \leq \bar{h}^{NF}$

3 For $\beta < 1 + \frac{r}{B}$, $\bar{x}^F \geq \bar{x}^{NF}$ and $\bar{h}^F \leq \bar{h}^{NF}$

4 For $\beta > 1 + \frac{r}{B}$ the steady state is stable and it holds that: $\bar{x}^F \leq \bar{x}^{NF}$ and $\bar{h}^F \geq \bar{h}^{NF}$

These inequalities follow also easily from a direct comparison of the systems of equations that determine the steady state in both cases, (C.14) to (C.17), (44) and (45) and (C.18) to (C.21), (50) and (51).
Apply proposition 5.4 to find that, for $\beta = 1$,

$$h^F(t) < h^{NF}(t), \tag{56}$$

since the condition in the proposition is met. This can, like the other results, also be checked directly by looking at equations (44) and (50).

4 Non-cooperative equilibrium

Consider the dynamic noncooperative game between the two countries. Each player is assumed to have complete information about the system dynamics and their interactions. We consider here only open-loop solutions. Let λ^f, λ^c, μ^f and μ^c be the costate variables for the country with and without forests respectively.

4.1 The net emissions case

Necessary conditions for a noncooperative Nash equilibrium solution in the open-loop game are

$$U'(e^f) = -\lambda^f \tag{57}$$

$$R'(h) = \mu^f - \lambda^f \tag{58}$$

$$\dot{\lambda}^f = (r+\alpha)\lambda^f + D'(Q) \tag{59}$$

$$\dot{\mu}^f = (r - F'(x))\mu^f + \beta\lambda^f F'(x) \tag{60}$$

and

$$U'(e^c) = -\lambda^c \tag{61}$$

$$\dot{\lambda}^c = (r+\alpha)\lambda^c + D'(Q) \tag{62}$$

$$\dot{\mu}^c = (r - F'(x))\mu^c + \beta\lambda^c F'(x) \tag{63}$$

In contrast to the cooperative solution, each country has private shadow values, measured as the costate variables, λ^f, λ^c, and μ^f, μ^c, for the two stocks involved. The development of the forest stock is determined solely by the country that owns the forest. The other country takes its growth or decline as given. Its valuation for this stock, μ^c, does not affect its decisions and is 'redundant' according to the definition of redundancy given in Dockner et al. (1985).
For this symmetric case it can, like in the cooperative optimum, be concluded from the first-order conditions (specifically from (57), (59), (61) and (62)) that in equilibrium $e^f = e^c = e$. Assume an equilibrium that converges to a steady state for the moment. An analysis of stability requirements is given in appendix E. Denote Nash equilibrium values by a superscript N. Integrate (59) or (62) forward and insert in (57) respectively (61) to find the net present value expressions:

$$U'(e^{iN}(t)) = \int_t^\infty e^{-(r+\alpha)(s-t)} D'(Q^N(s))dt \text{ for i=f,c.} \tag{64}$$

Comparison of expression (64) with its counterpart for the case of cooperation (23) shows that, for any expected time path of carbon in the atmosphere, energy use in the open-loop Nash

equilibrium is larger than in the cooperative optimum, for both countries. This reflects the well-known result that free-rider incentives prohibit noncooperative countries to reach a Pareto optimal outcome.
Equilibrium harvest rates satisfy the following net present value expression that is obtained from forward integration of (60) using (58).

$$R'(h^N(t)) = \int_t^\infty e^{-(r+\alpha)(s-t)} D'(Q^N(s))ds - \int_t^\infty e^{-r(s-t)+\int_t^s F'(x^N(u))du} \lambda^f \beta F'(x^N(s))ds \quad (65)$$

This is the counterpart of (24). The forest country decides on harvest rates and takes only home damage into account, reflected by the direct damage effect $D'(Q)$ and the indirect growth effect, where λ^f reflects the home valuation for marginal changes in the stock of greenhouse gases. Because only home damage is taken into account, the direct damage effect is considered to be smaller, which leads to higher harvest rates. Also λ^f will be smaller in absolute value than λ in case of cooperation, so that the same indirect growth effect causes a smaller change in the harvest rate. If stock effects are such that both under cooperation and in Nash equilibrium a higher rate of harvest increases growth and hence contributes to abatement (that is, $F'(x) < 0$), cooperation results in a higher harvest rate than the Nash equilibrium. Both terms of (65) are positive and smaller than the analogous terms of (41). Results are more ambiguous, however, when $F'(x) > 0$ for some t, since then cases may be thought of where $h^{F,N}(t) < h^{F,C}(t)$ for some t (here $h^{F,C}$ denotes the value of harvest in a cooperative solution that takes net emissions into account.). These results are another variant of the general result that noncooperative equilibria are inferior to cooperation with side payments.
The steady-state equilibrium conditions for the game are:

$$F(\bar{x}) = \bar{h} \quad (66)$$

$$\alpha\bar{Q} = (1-\beta)F(\bar{x}) + 2\bar{e} = (1-\beta)\bar{h} + 2\bar{e} \quad (67)$$

$$(r+\alpha)\bar{\lambda}^f = -D'(\bar{Q}) \quad (68)$$

$$(r - F'(\bar{x}))\bar{\mu}^f = -\beta F'(\bar{x})\bar{\lambda}^f \quad (69)$$

and

$$(r+\alpha)\bar{\lambda}^c = -D'(\bar{Q}) \quad (70)$$

$$(r - F'(\bar{x}))\bar{\mu}^c = -\beta F'(\bar{x})\bar{\lambda}^c \quad (71)$$

4.2 Comparisons with the gross emissions case

First-order conditions and steady-state equations for the Nash equilibrium in case of gross emissions are obvious analogues to equations (57) to (63) and (66) to (71) and therefore omitted here. They can be found in appendix B.

First compare the steady-state levels in the Nash equilibrium and in the cooperative optimum, both for the model that includes forest-atmosphere interactions (denoted by superscript F) and for the model that neglects these (denoted by superscript NF). It can be concluded that for both models, cooperation results in a lower steady-state stock of greenhouse gases and lower emissions than the Nash equilibrium. Formally:

Proposition 5.5 $\bar{e}^{NF,C} < \bar{e}^{NF,N}$; $\bar{Q}^{NF,C} < \bar{Q}^{NF,N}$; $\bar{e}^{F,C} < \bar{e}^{F,N}$; $\bar{Q}^{F,C} < \bar{Q}^{F,N}$.

These inequalities follow immediately from a comparison of the relevant steady-state conditions. They are another restatement of the result that noncooperative behaviour leads to suboptimal equilibria, with too high emissions and too much pollution from the point of view of a social planner.
A comparison of energy use with and without the inclusion of carbon-sequestration in forest decisions gives an analogue to proposition 5.1 for the noncooperative equilibrium:

Proposition 5.6 $\bar{e}^{NF,N} \leq \bar{e}^{F,N}$

The proof is an analogue to the one for proposition 5.1 and omitted. In a Nash-equilibrium steady state, both the country that owns the forest and the country without forest burn more fossil fuels in the net emissions case than in the gross emissions case. The country without forest can be said to 'use' part of the abatement capacity provided for by the forest country. Note that this is efficient. One of the countries would act suboptimal if only the forest country would increase fossil fuel use compared to the gross emissions equilibrium. That fossil fuel use in equilibrium is equal in the two countries is due to our symmetry assumptions.
The optimal harvest rate of forests in the model that neglects forest-atmosphere interactions (the gross emissions case) is only determined by the direct utility that the forest country derives from the harvest. Therefore, it is equal in the cooperative optimum and in the Nash equilibrium solution. This conclusion depends on our assumption of separable utilities.

Proposition 5.7 $\bar{h}^{NF,C} = \bar{h}^{NF,N}$ *and* $\bar{x}^{NF,C} = \bar{x}^{NF,N}$

This equality implies that we have for the Nash equilibrium the same benchmark steady-state values for x in the gross emissions case as in the cooperative equilibrium, namely x^1, x^r and x^2. An analogue to proposition 5.2 is then easy to formulate:

Proposition 5.8 *Given that $\beta = 1$, and $h^r > h^0$, there exists a steady state to the forest model that is characterized by: $x^2 < \bar{x}^{F,N}$ and $\bar{h}^F < h^0$. Furthermore, in this case, the forest model has at most two other steady states that are characterized by $x^1 < \bar{x}^{F,N} < x^r$ and $\bar{h}^F > h^0$.*

The proof to this proposition is very similar to the proof to proposition 5.2 and therefore omitted.
Now compare steady-state harvest rates in the Nash equilibrium and in the cooperative outcome, for the case of net emissions. Only the case of $\beta = 1$ will be considered. For large enough initial values of x (see proposition 5.2), $x^2 < \bar{x}$ both in the Nash equilibrium and in the

cooperative outcome. Then larger steady-state harvest rates imply higher forest growth, but lower steady-state forest stocks and larger (negative) marginal growth rates (that is, smaller in absolute value). In the Nash equilibrium, the forest country underestimates both the damage effect of the direct contribution of harvest to carbon in the atmosphere and the indirect effect of a smaller forest stock with larger marginal growth rate compared to the social optimum. Both work in the same direction and the result is that harvest is too large in the Nash equilibrium:

Proposition 5.9 *Given that $\beta = 1$, and $h^r > h^0$, for large enough initial value of x, $x^2 < \bar{x}^{F,N} < \bar{x}^{F,C}$ and $h^0 > \bar{h}^{F,N} > \bar{h}^{F,C}$.*

This proposition is proved in appendix E. For lower initial values of x, $x^1 < \bar{x}^{F,N} < x^r$ and $x^1 < \bar{x}^{F,C} < x^r$. A larger steady-state harvest rate implies more growth and sequestration, a larger stock of forest, higher direct contributions from harvest to carbon accumulation and a lower marginal growth rate. Again, in the Nash equilibrium, the forest country underestimates both the damage effect and the sequestration effect. A larger harvest rate implies a larger forest stock with smaller marginal growth rate compared to the social optimum. Now, however, these two effects do not work in the same direction. The damage effect would imply larger Nash-equilibrium harvest rates compared to the social optimum. The sequestration effect would point to a smaller stock with larger growth rates and hence larger harvests in the Nash equilibrium than in the social optimum. The overall effect is not clear ex ante and it is possible that $\bar{h}^{F,N} > \bar{h}^{F,C}$ with $x^1 < \bar{x}^{F,C} < \bar{x}^{F,N} < x^r$, or that the reverse holds and $\bar{h}^{F,N} < \bar{h}^{F,C}$ with $x^1 < \bar{x}^{F,N} < \bar{x}^{F,C} < x^r$.

A comparison of equilibrium paths is restricted to the case of $\beta = 1$. An analogue to proposition 5.4 can be formulated:

Proposition 5.10 *In the Nash equilibrium, let $e^{min,N} < e(t)$ and $Q^{min,N} < Q(t)$, where $e^{min,N}$ and $Q^{min,N}$ satisfy $(r + \alpha)U'(e^{min,N}) < D'(Q^{min,N})$. Then, for $\beta = 1$, for all t, $h^{F,N}(t) < h^{NF,N}(t)$.*

The proof, which is an adaption of the proof to proposition 5.4 is given in appendix E.

4.3 Example

The results for the noncooperative equilibrium can again be illustrated by the example. For that example, the game between the two countries belongs to the class of tractable games studied by Dockner et al. (1985). In particular, the game is control-state separated with respect to dynamics and objectives. This is mainly due to the simplifying assumptions of linear growth of forests and linear damage that were made in the example. As a result, the example allows for explicit analytical solutions of optimal paths in the cooperative case and equilibrium paths in the game. Moreover, the equilibrium feedback solution coincides with the open-loop equilibrium in this specific example.

In the steady state, the Nash equilibrium values of the control values are given as:

$$\hat{h}^{F,N} = \frac{\phi}{\psi} + \frac{d(B(\beta - 1) - r)}{\psi(r + \alpha)(r + B)} \tag{72}$$

and

$$\hat{e}^{i,F,N} = \frac{\chi}{\gamma} - \frac{d}{\gamma(r + \alpha)}. \tag{73}$$

Equilibrium time paths are given by

$$Q^N(t) = c_{1N}e^{-\alpha t} + c_{2N}e^{-Bt} + \bar{Q}^{F,N} \tag{74}$$

$$x^N(t) = (x_0 - \bar{x}^{F,N})e^{-Bt} + \bar{x}^{F,N} \tag{75}$$

$$\lambda^i(t) = \bar{\lambda}^{i,F,N}, \quad i = f,\, c \tag{76}$$

$$\mu(t)^i = \bar{\mu}^{i,F,N}, \quad i = f,\, c \tag{77}$$

with $c_{1N} = Q_0 - \bar{Q}^{F,N} - c_{2N}$ and $c_2 = \frac{\beta B(x_0 - \bar{x}^{F,N})}{\alpha - B}$. Apply proposition 5.5, 5.7 and 5.9 to compare the Nash equilibrium with the cooperative solution or alternatively directly compare the steady-state expressions. It follows that in Nash equilibrium, the gross emission of CO_2 by each country is higher than its cooperative emission (compare equation (45) and (73)).

From the equations that define $\hat{e}$ it is easily seen that for high discount rates the difference between Nash equilibrium and cooperation is low. The difference is more prominent if the natural removal of the carbon dioxide from the atmosphere is slow, as determined by a low value of the parameter α, or if marginal environmental damage is large.

From (44) and (72) it follows that for $B(\beta - 1) - r < 0$, or equivalently $\beta < 1 + \frac{r}{B}$, the noncooperative steady-state harvesting of the woods is more intensive than the cooperative harvesting. In this simple example, that holds irrespective of the initial value of x. For $\beta = 1$, alternatively proposition 5.9 may be applied, to find that $x^2 < \bar{x}^{F,N} < \bar{x}^{F,C}$ and $h^0 > \bar{h}^{F,N} > \bar{h}^{F,C}$, which of course again says that harvesting is more intensive in the noncooperative than in the cooperative steady state.

The difference between harvest in the Nash equilibrium and the cooperative solution is small when the marginal damage in the country without forest (which is the damage that is neglected in (72)) is small, or when $B(\beta - 1) - r$ is small in absolute value. From equations (44) and (72), it can be seen that for $\beta = 1$, when the discount rate, r, α or B is small then the difference between cooperative and noncooperative harvesting is large.

Since, for $\beta = 1$, both harvesting and emissions are higher in the noncooperative game the ultimate stock of the carbon dioxide in the atmosphere will be higher as well (see also equation (C.14) and (C.22)). Summarizing, in noncooperative equilibrium, neither player takes the foreign damage into account in their decisions, which results in higher levels of pollution and, for $\beta = 1$, in lower levels of forest, as compared to the cooperative game.

Compare to the game that neglects carbon sequestration. The first-order conditions and steady-state values of the variables are not explicitly written down for the Nash equilibrium, since they are variants of the cooperative equilibrium. From proposition 5.8, or a comparison of the relevant steady-state equations for the 'forest' and the 'no forest' case, follows that the steady-state stock of forests is larger when the role of forests in carbon dynamics is taken into account than when it is not, while the equilibrium harvest rate is lower.
As in the cooperative equilibrium, due to constant marginal damage, an equilibrium in the gross emissions case is characterized by monotone approximation of greenhouse gas stocks to their steady-state levels.
When the sink capacity is included, so the net emissions case is considered, equilibrium growth paths of the stock of greenhouse gases are no longer monotonic even given constant marginal damage. The sink capacity of forests may for some time-interval turn an increasing pollution path in a decreasing one. This sink capacity decreases, however, as forests increase and growth rates decline. This reflects the time-buying aspect of the carbon-sequestration by forests.

5 Conclusions

A model was set up to analyse the effects for emission-reduction requirements and forest growth when the net rather than the gross emissions of carbon dioxide from a country are counted. When the carbon dioxide abatement by the forests in a country is explicitly taken into account, this implies for the world as a whole both a potential additional source of carbon from forest harvest and a potential additional abatement opportunity from forest growth.
A comparison of cooperative optimal paths of forest harvest with net and gross emissions shows that for a certain range of parameter values, in case of net emissions, harvest is lower and forests are larger than when the interaction of forests with the carbon cycle is neglected. As a result, equilibrium use of fossil fuels can be higher. Similar results follow from a comparison of noncooperative open-loop equilibria with net and gross emissions.
These results are stated in harvest rates and as such reflect the simplification that was made in the modelling of forest growth and harvest. To find precise results in terms of rotation periods, a rotation period model would be needed. But roughly speaking, it could be said that lower harvest rates and larger volumes are obtained by larger rotation periods. Then considering net emissions implies an increase in rotation periods, which is confirmed by results from rotation period models (see Sedjo et al., 1995).
An interesting effect here is that in the steady state both countries have higher use of fossil fuels in the net emissions case than in the gross emissions case. Not only the country with the forests profits in terms of welfare from the increased carbon sequestration by its forest in the net emissions case, but the other country as well. It would be interesting to analyse the effects on bargaining on emission reduction efforts. It could for instance be analysed which country gains most from consideration of net emissions rather than gross emissions case. It seems that it is the country without forest, since this country does not have to adjust its harvest of forest, but may increase its use of fossil fuels.

The usual effect, that free-rider behaviour under transboundary pollution cause noncooperative emission levels to be higher than the global optimum, is also present when net emissions are counted. In the steady state, carbon dioxide emissions under cooperation are lower than that under noncooperation, the harvest rate is lower, the stock of forests larger and the stock of carbon dioxide smaller. A simple example shows that (even if inclusion of forest in the carbon dioxide emission reduction game does not change this general result) the interference of forest and carbon dynamics leads to more complex equilibrium paths.

In this chapter, harvested wood is considered to result in revenues separated from the utility derived from energy use. An interesting extension for further research would be to include the possibility that (some) wood is used as a substitute for fossil fuels. Tahvonen (1995) analyses an optimal growth model that includes this option. He considers the question what is the right way for a single country to implement an optimal carbon policy. In contrast, this chapter focuses on what would be the optimal policy target given that forests are important. When wood is a substitute for fossil fuels it may be expected that the carbon abatement effect of forest harvest is enlarged, because now additional harvest of wood implies that less fossil fuels have to be used. Thus it may be expected that equilibrium harvests are larger than the equilibrium harvest which resulted in the case that was analysed here.

A Stability

The net emissions case

A steady state of the cooperative problem is saddle-point stable when the modified Hamiltonian system that defines $(\dot{Q}, \dot{x}, \dot{\lambda}, \dot{\mu})$ as a function of (Q, x, λ, μ) has two eigenvalues with negative real parts and two eigenvalues with positive real parts. The modified Hamiltonian system is found from (4), (5) and (18), (19) with (16) and (17) inserted, to substitute for $\hat{e}$ and $\hat{h}$. From Feichtinger et al. (1994) it follows that the conditions $K \leq 0$ and $0 <$ det(J) are sufficient, but not necessary to guarantee saddle-point stability. Here det(J) denotes the determinant of the Jacobian matrix, given by

$$\begin{bmatrix} -\alpha & -\beta F'(\bar{x}) & 2\frac{-1}{U''} + \frac{-1}{R''} & \frac{1}{R''} \\ 0 & F'(x) & \frac{1}{R''} & -\frac{1}{R''} \\ 2D'' & 0 & r+\alpha & 0 \\ 0 & (\lambda\beta - \mu)F''(x) & \beta F'(x) & (r - F'(x)) \end{bmatrix} \tag{A.1}$$

and K is defined by:

$$\begin{vmatrix} -\alpha & 2\frac{-1}{U''} + \frac{-1}{R''} \\ 2D'' & r+\alpha \end{vmatrix} + \begin{vmatrix} F'(x) & \frac{-1}{R''} \\ (\lambda\beta - \mu)F''(x) & r - F'(x) \end{vmatrix}. \tag{A.2}$$

K can also be written as $K = -\alpha(r+\alpha) + \frac{4D''}{U''} + \frac{2D''}{R''} + c1$ with $c1 = F'(r-F') + \frac{(\lambda\beta-\mu)F''}{R''}$. When

$$c1 \leq 0, \tag{A.3}$$

it follows that $K < 0$. The determinant of J, is given by the rather complex expression:

$$-\alpha(r+\alpha)F'(r-F')-\alpha(r+\alpha)(\beta\lambda-\mu)\frac{F''}{R''}+2D''[\frac{1}{R''}F'(1-\beta)(r-(1-\beta)F')+\frac{2}{U''}(F'(r-F')+\frac{F''}{R''}(\lambda\beta-\mu))] \tag{A.4}$$

which can be rewritten to:

$$\text{det(J)} = -\alpha(r+\alpha)c1 + 2D''[\frac{2}{U''}c1 + \frac{1}{R''}c2] \tag{A.5}$$

with $c2 = (1-\beta)F'(r+(\beta-1)F')$
Sufficient conditions for det(J) to be nonnegative are (A.3) and

$$c2 \leq 0. \tag{A.6}$$

For det(J) to be positive, at least one of the conditions should hold with inequality. Sufficient conditions for saddle-point stability are then:

$$F'(1-\beta)(r-(1-\beta)F') \leq 0, \tag{A.7}$$

$$F'(x)(r-F'(x)) < 0 \tag{A.8}$$

and

$$(\lambda\beta-\mu)F''(x) \geq 0, \tag{A.9}$$

where the first condition is equivalent to (A.6) and the latter two are sufficient for $c1 < 0$.
Conditions (A.7), (A.8) and (A.9) are hence sufficient for saddle-point stability. When condition (A.9) is satisfied, this implies that the Hamiltonian is concave, which ensures that the net present value expressions (23) and (24) indeed describe an optimum.

The gross emissions case

The model that neglects the role of forests in carbon dynamics splits into two optimal control models. A cooperative solution to the first one is characterized by the modified Hamiltonian system :

$$\dot{Q} = -\alpha Q + 2\hat{e}(\lambda) \tag{A.10}$$

$$\dot{\lambda} = (r+\alpha)\lambda + 2D'(Q) \tag{A.11}$$

with $\hat{e}$ defined by:

$$U'(e) = -\lambda. \tag{A.12}$$

A steady state to this system is saddle-point stable when the Jacobian matrix, J, has a positive and a negative eigenvalue, which is equivalent to det(J) < 0. Since det(J) is given by:

$$-\alpha(r+\alpha) + \frac{4D''}{U''}, \tag{A.13}$$

this condition is satisfied. Furthermore, the steady state is uniquely defined by

$$\alpha\bar{Q} = 2\hat{e}(\bar{\lambda}) \tag{A.14}$$

and

$$\bar{\lambda} = \frac{-2D'(\bar{Q})}{r+\alpha}. \tag{A.15}$$

Hence, the unique steady state $(\bar{Q}^{NF}, \bar{e}^{NF})$ is always saddle-point stable, given the assumptions made in section 2.

The second model has the modified Hamiltonian system:

$$\dot{x} = F(x) - \hat{h}(\mu) \tag{A.16}$$

$$\dot{\mu} = (r - F'(x))\mu \tag{A.17}$$

with $\hat{h}$ determined by

$$R'(h) = \mu. \tag{A.18}$$

The Jacobian matrix that belongs to this system is given by $\det(J) = F'(x)(r - F'(x)) - \frac{F''(x)R'(h)}{R''(h)}$. Any steady state is defined by:

$$F(\bar{x}) = \hat{h}(\bar{\mu}) \tag{A.19}$$

and

$$\bar{\mu}(r - F'(\bar{x})) = 0 \tag{A.20}$$

Since $F''(x) < 0$ and $R''(h) < 0$, these two equations have at most three solutions, denoted by x^1, x^2 and x^r, with x^1 and x^2 characterized by $\mu = 0$ and x^r by $F'(x^r) = r$. For the steady states x^1 and x^2, $\mu = 0$, so that these are defined by $R'(\bar{h}) = 0$. This implies that in those steady states, det(J)= $F'(x)(r - F'(x))$. Denote h such that $R'(h) = 0$ by h^0. Since the equation $F(x) = h^0$ has two solutions, let x^1 be defined by $F(x^1) = h^0$ and $F'(x^1) > 0$, while x^2 is defined by $F(x^2) = h^0$ and $F'(x^2) < 0$. Then in the steady state (x^1, h^0), $h^0 < h^r$ implies that $F'(x^1) > r$, so that det(J)< 0, while in the steady state (x^2, h^0) det(J)< 0 follows from $F'(x) < 0$ immediately. These steady states are both saddle-point stable. For $\bar{x} = x^r$, det(J)$= -\frac{F''R'}{R''}$. From $h^r > h^0$, $R'(h^r) < 0$ and hence det(J)> 0. From trace(J)$= r > 0$, it then follows that in the neighbourhood of (x^r, h^r), the system is unstable.

From a consideration of the phase diagram it can be seen that for $x(0) > x^1$, the optimal convergent path converges to $\bar{x} = x^2$. Along the path, $\hat{h}(t) = h^0$, while $x(t)$ follows as the solution of

$$\dot{x} = F(x) - h^0 \tag{A.21}$$

with x(0) as the initial value. For $x(0) < x^1$, an optimal convergent path converges to x^1 and this path is characterized by $h(t) < h^0$. That implies that $\mu(t) > 0$ along this path.

B Proofs to the propositions

Proof to proposition 5.1

From equations (16) and (25) it follows that $\bar{e}$ is defined by $U'(\bar{e}) = -\bar{\lambda} = \frac{2D'(\bar{Q})}{r+\alpha}$ for both the case of net and the case of gross emissions. To prove this proposition define a function $g(Q)$ by $g(Q) = U'^{-1}(\frac{2D'(Q)}{r+\alpha})$. That is, $g(Q)$ gives the level of fossil fuel use, e, that satisfies $U'(e) = \frac{2D'(Q)}{r+\alpha}$. Take the derivative of both sides of this expression to find $g'(Q) = \frac{2D''(Q)}{(r+\alpha)U''(g(Q))}$. From the assumptions follows then $g'(Q) < 0$.

The stationary level of fossil fuel use in the forest model (the case of net emissions) is defined by (see (25) to (28)): $\bar{e}^F = g(\bar{Q}^F)$ with $\bar{Q}^F = \frac{1}{\alpha}[2\bar{e}^F + \bar{h}^F(1-\beta)]$, while in the gross emissions case the stationary level of fossil fuel use is defined by: $\bar{e}^{NF} = g(\bar{Q}^{NF})$ with $\bar{Q}^{NF} = \frac{1}{\alpha}[2\bar{e}^{NF}]$. Or, inserting for $\bar{e}$, $\bar{Q}^F = \frac{1}{\alpha}[2g(\bar{Q}^F) + \bar{h}^F(1-\beta)]$ and $\bar{Q}^{NF} = \frac{1}{\alpha}[2g(\bar{Q}^{NF})]$. From $\bar{h} \geq 0$ and $\beta \leq 1$ then follows that $\bar{Q}^F \leq \bar{Q}^{NF}$ which proves the proposition, since from $g'(Q) < 0$ it follows that $\bar{e}^F \geq \bar{e}^{NF}$ is equivalent to $\bar{Q}^F \leq \bar{Q}^{NF}$.

Proof to proposition 5.2

Remember that $x^1 < x^r < x^2$ denote the three steady states of the model with gross emissions. Furthermore, h^0 is the steady-state harvest rate that belongs to x^1 and x^2, and that is given by $R'(h^0) = 0$, while $h^r = F(x^r)$ is the steady-state harvest rate that belongs to x^r. Consider the set of equations (25) to (28), (16) and (17) that are necessary conditions for a stationary solution to the cooperative problem.

Insert (26) and (25) in (17) and use $\beta = 1$, to find that for gross emissions:

$$R'(\bar{h}) = \frac{2D'(\bar{Q})}{r+\alpha}\frac{r}{r-F'(\bar{x})} \tag{B.1}$$

This equation then defines the steady-state harvest rate together with (16), (27) and (28). Since $F''(x) < 0$, equation $\bar{h} = F(\bar{x})$ has at most two solutions for any given $\bar{h}$ in the range of F(x) (that is, for any $0 \leq \bar{h} \leq F(x^h)$), a solution with a positive and a solution with a negative value for $F'(x)$.

First restrict attention to the set $\{x|F'(x) > 0\}$. Consider a possible $\bar{h}^F$ which is larger than h^0. This implies that $R'(\bar{h}^F) < 0$, so that (B.1) is only satisfied for $F'(\bar{x}) > r$. From $h^0 < h^r$ and $F''(x) < 0$, this implies that such a steady state is characterized by: $x^1 < \bar{x} < x^r$. The steady state is given as the solution of:

$$R'(\bar{h}) = \frac{2D'(\bar{Q})}{r+\alpha}\frac{r}{r-F'(\bar{x})}; \quad F(\bar{x}) = \bar{h}; \quad \text{and } F'(\bar{x}) > 0 \tag{B.2}$$

Note that there may exist more than one solution (or none) to this set of equations.

A possible $\bar{h}^F$ smaller than h^0 cannot exist. Such a steady state would imply $R'(\bar{h}) > 0$, which would require that $F'(\bar{x}) < r$. That would again require that $\bar{h} > h^r$, since $F''(x) < 0$, which contradicts $\bar{h} < h^0$.

Now consider the set $\{x|F'(x) < 0\}$. Again consider a possible $\bar{h}^F$ which is larger than h^0. This implies that $R'(\bar{h}^F) < 0$, so that (B.1) is only satisfied for $F'(\bar{x}) > r$. That cannot be true for $F'(x) < 0$. Hence, no steady state with $\bar{h} > h^0$ exists.

A possible $\bar{h}^F$ which is smaller than h^0 implies that $R'(\bar{h}) > 0$. Equation (B.1) then leads to: $F'(\bar{x}) < r$. This is always true for $F'(x) < 0$. Hence there exists a steady state with $\bar{h} < h^0$ and $F'(x) < 0$. From $h^0 < \bar{h}^F$ and $F''(x) < 0$, it follows that this steady state is characterized by: $x^2 < \bar{x}$. The steady state is given as the solution of:

$$R'(\bar{h}) = \frac{2D'(\bar{Q})}{r+\alpha}\frac{r}{r-F'(\bar{x})}, \tag{B.3}$$

$$F(\bar{x}) = \bar{h} \tag{B.4}$$

and

$$F'(\bar{x}) < 0 \tag{B.5}$$

Note that, given $F'(x) = 0$ for some $x = x^h$, there always exists exactly one solution to this set of equations. This follows since $R'(h)$ is strictly decreasing in h, $\frac{r}{r-F'(x(h))}$ is strictly increasing in h, when $x(h)$ is defined by $F(x) = h$ and $F'(x) < 0$ and finally $R'(h)$ and $\frac{2D'}{r+\alpha)}\frac{r}{r-F'(x(h))}$ can be shown to cross each other.

Proof to proposition 5.3

The localization of possible steady states follows from a similar reasoning as in the previous section. However, now the correct equations to consider are:

$$R'(\bar{h}) = \frac{2D'(\bar{Q})}{r+\alpha}\frac{r+(\beta-1)F'(x)}{r-F'(\bar{x})}, \tag{B.6}$$

$$F(\bar{x}) = \bar{h}, \tag{B.7}$$

and

$$F'(\bar{x}) > 0; \ \text{ or } \ F'(\bar{x}) < 0 \tag{B.8}$$

Note that for $F'(x) < 0$, $r + (\beta - 1)F'(x) < 0$ whenever $\beta > 1 - \frac{r}{F'(x)}$. For $F'(x) > 0$, given $\beta \geq 1$, $r + (\beta - 1)F'(x)$ is always positive. It follows that for $F'(x) < 0$, the right-hand side of equation (B.6) is positive for $\beta < 1 - \frac{r}{F'(x)}$ and negative for $\beta > 1 - \frac{r}{F'(x)}$. Moreover, for $r > F'(x) > 0$, this right-hand side is positive and for $F'(x) > r$, this right-hand side is negative for all $\beta \geq 1$.
Now try to solve for (B.6) and (B.7) with $F'(x) > 0$. A solution with $\bar{h} < h^0$ must have $F'(\bar{x}) > r$ which implies that the right-hand side of equation (B.6) is negative. This contradicts $\bar{h} < h^0$, hence such a solution cannot exist. A solution with $\bar{h} > h^0$ may be characterized by $F'(\bar{x}) > r$ as well, when $h^0 < \bar{h} < h^r$. In that case, both sides of equation (B.6) are negative and such a steady state may exist. The steady state is then characterized by: $h^0 < \bar{h} < h^r$ and $x^1 < \bar{x} < x^r$. Trying a solution with $\bar{h} > h^r$ again leads to a contradiction.
Similarly, try to solve for (B.6) and (B.7) with $F'(x) < 0$. For small enough β, that is for $\beta < 1 - \frac{r}{F'(\bar{x})}$, the right-hand side of equation (B.6) is positive and any solution $\bar{h}$ must satisfy $R'(h) > 0$. Hence it can be concluded that a steady state may exist that is characterized by: $\bar{h} < h^0$ and $\bar{x} > x^2$. For large β, $\beta > 1 - \frac{r}{F'(\bar{x})}$, any solution $\bar{h}$ must be characterized by $R'(h) < 0$. It then follows that a steady state may exist with: $\bar{h} > h^0$ and $x^r < \bar{x} < x^2$.

Proof to proposition 5.4

First define Q^{min} and e^{min} such that:

$$(r+\alpha)U'(e^{min}) \leq 2D'(Q^{min}) \tag{B.9}$$

This requires Q^{min} and e^{min} to be large enough. From (B.9) it follows that $\alpha U'(e^{min}) \leq 2D'(Q^{min})$. Then, assume that $e^{min} \leq e(t)$, that $Q(t) \geq Q^{min}$, that $0 \leq h(t)$ and that $0 \leq x(t) \leq x^{max}$, with x^{max} defined by $F(x^{max}) = 0$. Remember that in proposition 5.4 $\beta = 1$.

In the gross emissions case, the cooperative solution is $h^{NF}(t) = h^0$. Reasoning backward, it will be derived now that $h^F(t) < h^0$ for all t, if $\bar{h}^F$ satisfies $\bar{h}^F < h^0$.

Assume hence that $\bar{h}^F < h^0$ and suppose that $h^F(t) = h^0$ for some $t = t^*$. Then it can be shown that $\alpha U'(e(t^*)) < 2D'(Q(t^*))$: For $e(t^*) > e^{min}$ and $Q(t^*) > Q^{min}$, $U'(e(t^*)) < U'(e^{min})$ and $D'(Q(t^*)) > D'(Q^{min})$, so that condition (B.9) is sufficient for $\alpha U'(e(t^*)) - 2D'(Q(t^*))$ to be negative for all $e(t^*) > e^{min}$, $Q(t^*) > Q^{min}$.

From equation (41), if $\alpha U'(e(t^*)) - 2D'(Q(t^*)) < 0$ and $h^F(t^*) = h^0$, then $\dot{h}^F(t^*) > 0$. Hence, if it can be proved that $e^F(t) > e^{min}$ and $Q^F(t) > Q^{min}$ for all t, then if $h = h^0$ for some t^*, it follows that $\dot{h}^F > 0$. Reasoning backward then, it follows that if $h^F(t)$ 'comes close to' h^0, it will decrease and stay below h^0.

Summarizing, provided that $e^F(t) > e^{min}$ and $Q^F(t) > Q^{min}$, where Q^{min} and e^{min} satisfy (B.9), it can be proved that $h^F(t) < h^{NF}(t)$.

To prove that $x^F(t) > x^{NF}(t)$, reason forward. It is given that $x^F(0) = x^{NF}(0) = x(0)$. Use $d(t) = x^{NF}(t) - x^F(t)$ and take its time derivative, $\dot{d}(t) = F(x^{NF}(t)) - F(x^F(t)) + h^F(t) - h^{NF}(t)$. Use $h^F(t) < h^{NF}(t)$ which was just proved to see that $\dot{d}(0) < 0$. Suppose that $d(t) = 0$ for some $t = t^*$. Then again use $h^F(t) < h^{NF}(t)$ to see that $\dot{d}(t^*) < 0$ also. It follows that $d(t) < 0$ for all $t > 0$, which is equivalent to $x^F(t) > x^{NF}(t)$.

C Algebra in the example

This appendix summarizes some results for the example. The first-order conditions in case of net emissions are:

$$\phi - \psi h^F + \lambda - \mu = 0 \tag{C.10}$$

$$\chi - \gamma e^{iF} + \lambda = 0 \quad \text{i=f,c} \tag{C.11}$$

$$d\lambda/dt = (r+\alpha)\lambda + 2d \tag{C.12}$$

$$d\mu/dt = -\beta\lambda B + (r+B)\mu \tag{C.13}$$

Steady-state values of the variables for the case of net emissions:

$$\bar{Q}^F = \frac{1}{\alpha}[\frac{\phi}{\psi}(1-\beta) + \frac{2\chi}{\gamma} + (1-\beta)\frac{2d(B(\beta-1)-r)}{\psi(r+\alpha)(r+B)} - \frac{4d}{\gamma(r+\alpha)}], \tag{C.14}$$

$$\bar{\lambda}^F = \frac{-2d}{(r+\alpha)}, \tag{C.15}$$

$$\bar{\mu}^F = \frac{-\beta B 2d}{(r+\alpha)(r+B)} \tag{C.16}$$

and

$$\bar{x}^F = \frac{b}{B} - \frac{1}{B}[\frac{\phi}{\psi} - \frac{2d(B(\beta-1)-r)}{(r+\alpha)(r+B)\psi}]. \tag{C.17}$$

The modified Hamiltonian system can be written as $dV/dt = JV + W$ with $V' = [Q\ x\ \lambda\ \mu]$,

$$J = \begin{bmatrix} -\alpha & \beta B & \frac{1}{\psi}+\frac{2}{\gamma} & -\frac{1}{\psi} \\ 0 & -B & -\frac{1}{\psi} & \frac{1}{\psi} \\ 0 & 0 & r+\alpha & 0 \\ 0 & 0 & -\beta B & r+B \end{bmatrix}$$

and

$$W' = [(\frac{\phi}{\psi} - \beta b + \frac{2\chi}{\gamma});\ (b - \frac{\phi}{\psi});\ 2d;\ 0].$$

Steady-state values of variables for the gross emissions case are:

$$\bar{Q}^{NF} = \frac{1}{\alpha}[\frac{2\chi}{\gamma} - \frac{4d}{\gamma(r+\alpha)}], \tag{C.18}$$

$$\bar{\lambda}^{NF} = \frac{-2d}{(r+\alpha)}, \tag{C.19}$$

$$\bar{\mu}^{NF} = 0 \tag{C.20}$$

and

$$\bar{x}^{NF} = \frac{b}{B} - \frac{1}{B}\frac{\phi}{\psi} \tag{C.21}$$

Finally, the steady-state values of the Nash equilibrium solution under net emissions are given by:

$$\bar{Q}^{F,N} = \frac{1}{\alpha}[\frac{\phi}{\psi}(1-\beta) + \frac{2\chi}{\gamma} + (1-\beta)\frac{d(B(\beta-1)-r)}{\psi(r+\alpha)(r+B)} - \frac{2d}{\gamma(r+\alpha)}] \tag{C.22}$$

$$\bar{\lambda}^{i,F,N} = \frac{-d}{(r+\alpha)}, \quad i = f,\ c \tag{C.23}$$

$$\bar{\mu}^{i,F,N} = \frac{-\beta B d}{(r+\alpha)(r+B)}, \quad i = f,\ c \tag{C.24}$$

and

$$\bar{x}^{F,N} = \frac{b}{B} - \frac{1}{B}[\frac{\phi}{\psi} + \frac{d(B(\beta-1)-r)}{(r+\alpha)(r+B)\psi}]. \tag{C.25}$$

D Nonnegativity constraints for the example

For the functional forms chosen in the example, some nonnegativity constraints must be laid on the parameters in those functions. Else, the variables could possibly take values with no economic meaning. In this appendix, these constraints are derived.

First consider the case of a cooperative solution in the gross emissions case. The steady-state values of the variables are then given by equations (C.18) to (C.21) and (50) and (51). It should hold that the steady-state stocks of carbon and forests are both nonnegative. This implies that

$$\frac{\phi}{\psi} \leq b \tag{D.1}$$

and

$$2d \leq \chi(r+\alpha) \tag{D.2}$$

The latter condition also guarantees that the stationary rate of fossil fuel use given in (51) is nonnegative.

Second consider the case of a cooperative solution to a model that incorporates the carbon sequestration by forests, that is, the net emissions case. The steady-state values of the variables are then given by equations (C.14) to (C.17) and (44) and (45). Nonnegativity of the variables is satisfied if the following conditions hold

$$2d \leq \chi(r+\alpha) \tag{D.3}$$

$$2d(r+B(1-\beta)) \leq \phi(r+\alpha)(r+B) \tag{D.4}$$

$$2d(r+B(1-\beta)) \geq (\phi - b\psi)(r+B)(r+\alpha) \tag{D.5}$$

and

$$2d[(\beta-1)(r+B(1-\beta)) - \frac{2\psi}{\gamma}(r+B)] \geq (r+\alpha)(r+B)(\phi(\beta-1) - \frac{2\psi\chi}{\gamma}) \tag{D.6}$$

For the Nash equilibrium case, similar conditions can be derived. These are for the case of gross emissions:

$$d \leq \chi(r+\alpha) \tag{D.7}$$

and

$$\frac{\phi}{\psi} \leq b \tag{D.8}$$

For the case of net emissions, the nonnegativity conditions are:

$$d \leq \chi(r+\alpha) \tag{D.9}$$

$$d(r+B(1-\beta)) \leq \phi(r+\alpha)(r+B) \tag{D.10}$$

$$d(r+B(1-\beta)) \geq (\phi - b\psi)(r+B)(r+\alpha) \tag{D.11}$$

and

$$d[(\beta-1)(r+B(1-\beta)) - \frac{2\psi}{\gamma}(r+B)] \geq (r+\alpha)(r+B)(\phi(\beta-1) - \frac{2\psi\chi}{\gamma}) \tag{D.12}$$

E The noncooperative equilibrium

Stability

For stability analysis of the open-loop Nash equilibrium, we use results of Haurie and Leitmann and refer to Hämäläinen, Haurie and Kaitala (1985), for more details on these results. The results of Haurie and Leitmann give sufficient conditions for the global asymptotic stability of a steady state. The conditions require certain matrixes to be negative definite. Since multiple steady states cannot be excluded, we have to restrict this stability to hold only for initial values that lie in specific subsets. Thus separately consider the intervals $[0, x^1)$, (x^1, x^r), (x^r, x^2) and (x^2, ∞).

First consider the model that neglects carbon sequestration. This model can be split up in two models, like in the cooperative case. For the first model, a stationary equilibrium is defined by (E.15), (E.16) and (E.18) together with (E.7) and (E.11). A sufficient condition for the stationary point of this model to be stable is:

$$\frac{1}{4}r^2 < -\frac{D''(Q)}{U''(e^i)} \text{ for all nonzero } (e^i, Q), \text{ for i=c,f.} \tag{E.1}$$

This condition ensures that the relevant matrices are negative definite.

For the second model, a stationary equilibrium is defined by (E.14), (E.17) and (E.19) together with (E.8). Sufficient conditions for the stationary points to be stable are then:

$$\mu^f > 0 \tag{E.2}$$

$$\frac{1}{4}r^2 < \frac{F''(x)}{R''(h)}\mu^f \text{ for all nonzero } (h, x). \tag{E.3}$$

The first condition is equivalent to $h < h^0$ and hence restricts the range from which to choose h. This implies that for $x(0) \in (x^1, x^2)$, when it would be an equilibrium choice to have $h > h^0$, paths cannot be ensured to converge.

For the model that includes carbon sequestration, sufficient conditions for stability are:

$$\mu^f - \beta\lambda^f > 0 \tag{E.4}$$

$$D''(\frac{1}{R''} + \frac{1}{U''}) + \frac{1}{4}r^2 < 0 \tag{E.5}$$

$$\frac{1}{4}r^2\frac{D''}{R''} + (\frac{1}{4}r^2 + \frac{D''}{U''})(\frac{1}{4}r^2 - \frac{F''}{R''}(\mu^f - \beta\lambda^f)) > 0 \tag{E.6}$$

Equilibrium conditions for gross emissions

In a noncooperative Nash equilibrium, the following conditions must be satisfied:

$$U'(e^f) = -\lambda^f \tag{E.7}$$

$$R'(h) = \mu^f \tag{E.8}$$

$$\dot{\lambda}^f = (r + \alpha)\lambda^f + D'(Q) \tag{E.9}$$

$$\dot{\mu}^f = (r - F'(x))\mu^f \tag{E.10}$$

and

$$U'(e^c) + \lambda^c = 0 \tag{E.11}$$

$$\dot{\lambda}^c = (r + \alpha)\lambda^c + D'(Q) \tag{E.12}$$

$$\dot{\mu}^c = (r - F'(x))\mu^c \tag{E.13}$$

Steady-state equilibrium conditions are:

$$F(\bar{x}) = \bar{h} \tag{E.14}$$

$$\alpha\bar{Q} = 2\bar{e} \tag{E.15}$$

$$\bar{\lambda}^f = \frac{-D'(\bar{Q})}{(r + \alpha)} \tag{E.16}$$

$$\bar{\mu}^f = 0 \text{ or } \bar{\mu}^f = R'(h^r) \tag{E.17}$$

and

$$\bar{\lambda}^c = \frac{-D'(\bar{Q})}{(r+\alpha)} \tag{E.18}$$

$$\bar{\mu}^c = 0 \text{ or } \bar{\mu}^c = R'(h^r) \tag{E.19}$$

Proof to proposition 5.9

From proposition 5.5:

$$\bar{e}^{F,N} > \bar{e}^{F,C} \tag{E.20}$$

This is equivalent to $U'(\bar{e}^{F,N}) < U'(\bar{e}^{F,C})$ or $\bar{\lambda}^{i,F,N} > \bar{\lambda}^{F,C}$. Since $\beta = 1$, the stationary rate of harvest is given by the two equations

$$F(\bar{x}^F) = \bar{h}^F \tag{E.21}$$

and

$$R'(\bar{h}^F) = \frac{-\bar{\lambda}r}{r - F'(\bar{x}^F)} \tag{E.22}$$

both in the Nash equilibrium and in the Cooperative optimum.

From propositions 5.2 and 5.8, it is known that steady states either have $x^1 < \bar{x}^F < x^r$ or $x^2 < \bar{x}^F$ both in the Cooperative and in the Nash equilibrium.

Consider solutions to (E.21) that have $x^2 < \bar{x}^F$ for both types of equilibria. These solutions are relevant for large enough initial value of x. Then $F'(\bar{x}^F) < 0$. It will be proved by contradiction that

$$\bar{h}^{F,N} > \bar{h}^{F,C} \tag{E.23}$$

Assume that

$$\bar{h}^{F,N} < \bar{h}^{F,C} \tag{E.24}$$

That would imply $\bar{x}^{F,N} > \bar{x}^{F,C}$. This in turn implies that $0 < \frac{r}{r-F'(\bar{x}^N)} < \frac{r}{r-F'(\bar{x}^C)}$ Combined with $\bar{\lambda}^{i,F,N} > \bar{\lambda}^{F,C}$ that results in $\frac{-\bar{\lambda}^{i,F,N}r}{r-F'(\bar{x}^N)} < \frac{-\bar{\lambda}^{F,C}r}{r-F'(\bar{x}^N)} < \frac{-\bar{\lambda}^{F,C}r}{r-F'(\bar{x}^C)}$. From (E.22) then follows that $R'(\bar{h}^{F,N}) < R'(\bar{h}^{F,C})$. This is equivalent to $\bar{h}^{F,N} > \bar{h}^{F,C}$, which contradicts the assumption that $\bar{h}^{F,N} < \bar{h}^{F,C}$. Thus it must hold that $\bar{h}^{F,N} > \bar{h}^{F,C}$, since the reverse assumption results in a contradiction.

So for $\beta = 1$,

$$x^2 < \bar{x}^{F,N} < \bar{x}^{F,C} \text{ and } h^0 > \bar{h}^{F,N} > \bar{h}^{F,C}. \tag{E.25}$$

In the main text it was derived that for small initial values of x, such that $x^1 < \bar{x}^{F,N} < x^r$ and $x^1 < \bar{x}^{F,C} < x^r$ it cannot be said whether $x^N > x^C$ or vice versa.

This neglects the possibility that under the Nash equilibrium, the value of $\bar{x}$ is in another interval than under the cooperative equilibrium. There are two possible equilibria then: $\bar{x}^{F,N} > x^2$ while $x^1 < \bar{x}^{F,C} < x^r$ and the reverse case. For $\bar{x}^{F,N} > x^2$, $\bar{h}^{F,N} < h^0$. Since $x^1 < \bar{x}^{F,C} < x^r$ implies that $\bar{h}^{F,C} > h^0$, then we have the possible equilibrium:

$$x^1 < \bar{x}^{F,C} < x^r < x^2 < \bar{x}^{F,N} \text{ and } \bar{h}^{F,N} < h^0 < \bar{h}^{F,C} \tag{E.26}$$

The reverse case results in:

$$x^1 < \bar{x}^{F,N} < x^r < x^2 < \bar{x}^{F,C} \text{ and } \bar{h}^{F,C} < h^0 < \bar{h}^{F,N} \tag{E.27}$$

However, given equal initial values, it seems not so likely that one of these two equilibria would occur.

Proof to proposition 5.10

The proof is analogous to the proof for proposition 5.4, but should be adjusted, since the forest country only includes home damage. Thus adjust condition B.9 and define $Q^{min,N}$ and $e^{min,N}$ such that:

$$(r+\alpha)U'(e^{min,N}) \leq D'(Q^{min,N}) \tag{E.28}$$

Furthermore, instead of (41), the following growth rate should be used:

$$\frac{\dot{h}^{F,N}}{h^{F,N}} = \frac{R'(h)}{R''(h)h}(r - F'(x)) + \frac{\alpha U'(e) - D'(Q)}{R''(h)h} \tag{E.29}$$

which is obtained from differentiation of (58), inserting (59) and (60). Then, the proof is obtained through the same reasoning as given in the proof for proposition 5.4.

Chapter 6

Strategic trade and transboundary pollution: a dynamic model

This chapter analyses the interaction between international rivalry and transboundary pollution. Trade is modelled as oligopolistic competition between firms. Governments are assumed to be interested both in high profits for domestic firms and in low domestic environmental damage. As a result there is strategic interaction between firms, between governments and between firms and governments.
A differential game model is used to analyse a duopoly, with each competitor situated in a different country. If governments impose the strict policy they prefer from an environmental point of view, this might harm domestic firms too much and decrease their profits substantially. It is assumed that governments do not want this, since they have a preference for high profits to be earned at home. This leads to downward distortions in environmental policy. Transboundary pollution is another source of distortions. This chapter analyses these distortions for the policy instruments taxes and standards.

1 Introduction

Transboundary pollution implies that emissions originating in one country cause environmental damage in another country. If regulators only care about damage in their own country, then pollution in other countries is no incentive for these regulators to adjust their environmental policy. From the point of view of all countries, environmental policies are too lax when national regulators neglect damage abroad that is due to domestic emissions. If the countries cooperate, they impose stricter regulations (See for example, Mäler, 1989).
Competition between domestic and foreign firms in an oligopolistic market is a second reason for governments to distort environmental policy. Distortions in environmental policies may namely act as commitment devices for firms and give them a strategic advantage towards their competitors. The phenomenon that governments set laxer environmental policy for trade strategic reasons is called 'environmental dumping' (See Rauscher (1996)).

With transboundary pollution, countries may have an even stronger incentive for environmental dumping. Foreign emissions add to domestic pollution levels, and a strict environmental policy at home might increase the market share of foreign relatively dirty firms, or lead to home firms relocating to areas with laxer environmental policy. In this way the condition of the environment may be made even worse[1].

The aim of this chapter is to analyse why and in what direction governments distort their environmental policies if both transboundary pollution and strategic trade are present. Governments are assumed to have an interest to improve the position of their home firms, but also a desire for a good environmental quality. They must strike a balance between these aims. A comparison is made between the resulting distortions in the policy instruments emission taxes and emission standards.

Trade is modelled as international competition between two firms, each located in a different country. These firms produce output, which they sell on the international market. They use a polluting input in their production processes.

Only pollution related to production processes is considered. Another type of pollution is related to products. These products cause pollution in the country where they are consumed and disposed of and not in the country that produces them. This type of pollution is left out of consideration. Trade policies, for instance import restrictions for products that have been produced through a polluting process, are ignored too. This type of policies might conflict with free trade agreements. However, a change is observed, because such agreements now tend to take environmental concerns into account (See for instance the discussion on NAFTA by Esty, 1994).

A dynamic model is used to describe a firm that can invest in production with less pollution and save production costs with that investment. By assumption, once the firm has invested in abatement, it has gained a competitive advantage. But without environmental policy, the net present value of such an investment is negative. Therefore, in the absence of environmental policy, the firm would not invest in abatement. For an analysis that includes the possibility that this net present value is positive, see Gabel and Sinclair-Desgagné (1997). The assumption that investment in abatement is rewarding (though not so rewarding that firms invest voluntarily) is made to incorporate the idea that a reduction in emissions usually implies a more efficient production in the sense of less inputs per unit of output. Due to this assumption the model differs from Kennedy (1994), where abatement is a pure cost to firms. A formulation comparable to ours is used in Kort (1994), Ulph and Ulph (1996) and Ulph (1994). The latter two papers use a multistage game, while Kort considers abatement investment in a dynamic model as a flow rather than a stock variable.

Compared to a multistage game, a full dynamic analysis in a differential game model is interesting, because it allows for the analysis of different equilibria, with strategies that differ in the degree of commitment, which is the key aspect of the effect on strategic interaction. The feedback Stackelberg equilibrium is a type of equilibrium that describes a plausible situation. An equivalent equilibrium in a multistage model does not exist. Furthermore, in a dynamic

[1] In the context of global warming, this is also referred to as 'carbon leakage'.

game model the behaviour of players at all points in time can be analysed, rather than just the steady state.

First a formalization of transboundary pollution is given in section 2. Then, section 3 describes the motives that may lead regulators to distort environmental policy from its socially optimal level in a situation with transboundary pollution and international competition. A differential game model is formulated in section 4 to describe the decision problems faced by firms and regulators. The next section, 5, discusses three scenarios, respectively a situation of full cooperation, a situation with a cooperative government that regulates competing firms and a situation where both firms and regulators compete on the international market. Section 6 concludes the chapter.

2 Transboundary pollution

Transboundary pollution is an important aspect of the analysis in this paper. Since for a lot of environmental problems eventually stocks of accumulated pollution cause environmental damage, ideally environmental objectives should be stated in terms of these stocks[2]. Here, for simplicity reasons, it is assumed that the objectives can be stated in terms of flows of pollution. That is, it is assumed that the regulators in each country, i, have an environmental damage function, $D^i(.)$, to value deposition at a given time. Marginal damage is assumed to be non-decreasing in depositions, $D^{i\prime\prime}(.) \geq 0$. Countries that value pollution differently or that differ in natural characteristics have different damage functions.

The following linear equation is used to formalize the dispersion of emissions over the countries (see for example Mäler, 1989). Let depositions in country i be given by P^i, emissions from the firm in country i by e^i, and emissions from the firm in country j by e^j, then

$$P^i = \beta^{ii} e^i + \beta^{ij} e^j \tag{1}$$

where pollution from elsewhere is abstracted away. β^{ij} is an element of a transportation matrix and denotes the percentage of emissions from one country that is deposited in the other country. Given one unit of emissions in country j, a fraction β^{ij} of it is deposited in country i and β^{jj} stays in country j. It is assumed that $\beta^{ij} + \beta^{jj} \leq 1$, since some of the pollution may go to a third country. This linear formulation contains a number of scenarios. For instance, in the absence of transboundary pollution, when pollution is purely local, $\beta^{ij} = \beta^{ji} = 0$. This case is shortly mentioned in section 5, where it is compared with the case of transboundary pollution.

[2] See for example van der Ploeg and de Zeeuw (1992) for a dynamic analysis with environmental damage from stocks.

3 Reasons to distort policy

Pigou concluded that it is optimal for regulators to set environmental policy such that marginal social costs are equated to marginal social benefits (Pigou, 1932). The standard analysis of environmental policy instruments is in a closed economy setting. In a framework where economies interact, several effects may cause distortions in environmental policy.

In the literature[3] distortions that are due to transboundary pollution and distortions that have to do with trade and international competition can be found. In case of transboundary pollution a prisoner dilemma type of game is played among governments with the common objective of a clean environment, but the competing objectives of low abatement costs. Free-rider incentives cause governments to set laxer-than-optimal policies and to neglect their influence on depositions in the other country. When trade is characterized by oligopolistic competition between domestic and foreign firms, international rivalry may lead countries to reduce the severity of their environmental policies, as a substitute for tariffs and other trade policy.

The last effect may be relatively small in case of local pollution, for the government that 'misuses' its environmental policy for trade strategic reasons has to pay for this with more environmental damage. In case of transboundary pollution, however, the distorting effect is strengthened. The additional damage is partly deposited in foreign countries and hence is neglected by the home country, while a decrease in foreign output decreases home damage. So transboundary pollution and trade strategic considerations are interdependent reasons to distort environmental policy from the level that is optimal from the point of view of all countries together.

These reasons for distortion will be sorted out in more detail below and given a number for later reference. First, the reasons connected with transboundary pollution are the following: a first reason for distortion is that countries only take local damage into account. Damage abroad is a part of global social costs, but not reckoned as such by a national government. This leads to a too lax environmental policy (1). Another distortion occurs, since countries want to shift costs of pollution reduction to other countries. If the state of the environment influences the decisions of the other country, a bad environment may stimulate foreign countries to more abatement. That may lead countries to set too lax a policy, with efforts of the others partly offsetting the distortion in environmental policy. This second distortion is absent in the analysis below, because here the valuation of damage is related to flows of pollution rather than stocks.

Second, consider the reasons for distortion that are due to imperfect competition. If domestic firms compete with foreign firms, there is reason for strategic distortion of environmental policy, since the choices of foreign firms can be indirectly influenced by domestic policy. A lax environmental policy in the home country stimulates domestic production, partially at the cost of foreign production. So some output is shifted from abroad (while total output increases). As a result, profits of the home firm increase. Three reasons for distortion can be distinguished. First, since governments want to shift rents from foreign to domestic firms, they distort their environmental policy (2). The type of distortion depends on the structure of the output market. In this chapter, we restrict our analysis to Cournot competition, and environmental policy is

[3] Part of this literature is reviewed in the introductory chapter 1.

distorted downwards. An extension to Bertrand competition is straightforward (See Barrett, 1994) and would imply upward distortion in environmental policy. Second, countries want to reduce environmental damage from abroad, hence they want to discourage production abroad. This is another reason to set their own policy too lax (3). Finally, countries do not take into account that a decrease in their own firm's output (and profits) increases the profitability of the foreign firm's production. This is the usual effect of oligopoly. If the firms would form a cartel and cooperate they could earn more profits. Given the objective function, that would be welfare improving[4]. In short, governments overestimate the total (global) social marginal costs of emission reductions by their home firm and set policy too lax (4). These 4 reasons for distortion will be present in the analysis below. They all point into the direction of a laxer environmental policy than is socially optimal from the point of view of the two countries together.

When consumer surplus is explicitly taken into account, an additional distortion can be distinguished. In case of oligopolistic competition, consumers favour increased output and hence lax environmental policies. Since the model below abstracts from consumer surplus a good discussion of this effect is not possible. In the conclusions we come back to the implications thereof.

Transboundary pollution is for instance analysed by Conrad (1993), Katsoulacos et al. (1996), Kennedy (1994) and van der Ploeg and de Zeeuw (1992). Distortions due to trade policy objectives are the subject of a large number of papers (for example, Barrett (1994), Conrad (1993), Katsoulacos et al. (1996), D.Ulph (1994)). A good overview is provided by A.Ulph (1994). Consumer surplus is included for example by Kennedy (1994) and Ulph (1996a).

4 An international duopoly

In this section a differential game model is formulated to analyse the distortions mentioned above. The model describes two countries with home firms that compete with each other on a world market. Governments are assumed to set environmental policy to maximize an objective function that values firm profits and reductions in the environmental damage caused by the emissions from that firm and its foreign competitor. Consumer surplus is neglected, that is, it is assumed that the largest part of the consumers is located outside the two countries. Two environmental policy instruments are discussed: a tax and an emission standard. Both countries are assumed to apply the same environmental policy instrument. Firms are modelled as profit maximizers. They compete with each other on the market for outputs. The analysis below is only a partial analysis. Effects on other sectors of the economy are left out, as well as adjustments in other than environmental policies. To sum up, firms decide on their output and investments in emission reduction and governments decide on environmental policy.

[4]Of course, usually consumer surplus is also included into welfare so that cartels are not better than oligopolies from a welfare point of view. In the current setting the governments are solely considering environmental damage and firm profits.

Investment decisions differ from decisions on environmental policy and output, because their effects last longer. Investment builds a capital stock, that affects future profits, while output and environmental policy decisions only influence the current period.

A country's government by assumption acts as a Stackelberg leader vis a vis the firm in its country. This is taken here to mean that the government's decisions at some point in time are given to the firm, when it has to take its decisions. It is more reasonable to assume that the firm knows the decision of the government and takes this as given, than the reverse. Reasons for that are the higher degree of openness and publicity of government decisions when compared to firm decisions. Furthermore, the government can bind itself with the help of laws. The length of the period of credible commitment for the government however, is an open question. It depends on the practise of law setting and the reputation of the government. If the government is able to change its policy quickly and can commit for only a short period, a feedback equilibrium is the proper equilibrium concept. This results in a solution that consists of strategies that are contingent on the situation at hand. The equilibrium strategies are time consistent and subgame perfect. If the government can commit for a long period, it is more appropriate to apply an open-loop equilibrium. Note that the structure of multistage models such as analysed in Kennedy (1994) and Ulph and Ulph (1996) is similar to a dynamic game model with an open-loop structure. In these settings, the government sets its policy once and for all and cannot react to firm decisions.

It is assumed that firms set output according to a feedback strategy. That is, firms' commitment to their output decisions is assumed to be short. They can adjust their rate of output instantaneously without adjustment costs. This is an abstraction, but compared to investment decisions it seems reasonable to assume that it is easy to adjust output plans. If the investment decisions of firms are rigid and require long term commitments, firms could be modelled to compete in an open-loop equilibrium. If investment strategies can adjust quickly, firms' competition could be modelled by a feedback equilibrium. Here, both possibilities are considered.

The interaction between governments of different countries and the competition between firms is modelled as a (Cournot) Nash equilibrium. If countries can commit their policies for a long period, an open-loop Nash equilibrium in environmental policies between the two governments results. If not, the equilibrium is a feedback Nash equilibrium in environmental policies. Similarly, if firms apply feedback strategies for output and investment, a feedback equilibrium results, while if they apply open-loop investment strategies, the resulting equilibrium is open-loop in investment and feedback in output.

The interaction between a government and a firm is more complicated. At each point in time, each government is by assumption a Stackelberg leader for the firms, because the firms take the decisions of governments as given when they decide on output and investment rates. But the stock of abatement capital influences government decisions if the government applies a feedback strategy. The firm takes these influences into account, so that its investment decisions are affected by the effect that changes in the capital stock have on government regulation. Following Başar and Olsder (1995, proposition 7.7, p.418), the equilibrium solution is called a feedback Stackelberg solution as regards the interaction between a government and a firm, if firms and governments use feedback decision strategies.

If governments use open-loop decision strategies, the equilibrium is called an open-loop Stackelberg solution (cf. Başar and Olsder, 1995, p.410). In that case, governments decide on a time path for their environmental policy, taking into account the effect on the equilibrium investment strategies. The firms take government policy as given in their decisions.
The third possibility, where firms use open-loop investment strategies and governments use feedback strategies, is not consistent with the assumption that the firm knows the environmental policy before it takes its decisions, at each point in time.
For consistency, a government that uses open-loop strategies in the game with firms, is also assumed to be in open-loop equilibrium with the other country. If governments use feedback strategies in the game with firms, then they are assumed to be also in feedback equilibrium with the other country. Analogous consistency requirements hold for firms.
Hence, several equilibria exist, depending on the type of strategies assumed to be relevant for the decision variables output, investment and environmental policy. As mentioned output is always set according to a feedback strategy. For investment and environmental policy, the case where both follow open-loop strategies and the case where both follow feedback strategies are considered. The following section (section 4.1) first presents a formal model of the firm and the regulator. Then in section 5 three benchmark solutions are discussed, one where the two firms are a cartel and the governments cooperate, another where firms compete but governments cooperate and a third where both firms and governments compete. In each case both countries are assumed to apply the same environmental policy instrument. But countries may differ in the strictness of their policy.

4.1 A duopoly model

Consider a firm i that maximizes profits from exports over an infinite time horizon. Let its production capacity be fixed to focus on the adjustment of technology to environmental requirements. For production of output, x^i, the firm uses a polluting input, e^i. The world market price per unit of this input is p^e. Environmental policy as faced by the firm is a tax, τ^i, on the use of the polluting input e^i. Alternatively, a standard on emissions, in the form of a maximum, $\bar{e}^i$, on the use of the polluting input is considered.
The environmental friendliness of the production process is summarized by a variable A^i, referred to as 'abatement capital'. If the firm owns a stock A^i of abatement capital and wants to produce x^i of output, it needs to use $e^i = E(A^i)x^i$ of the polluting input. Hence it is assumed that the firm needs a fixed amount of $E(A)$ per unit of output. This amount can be reduced if the firm, through investment in abatement technology, increases its stock of abatement capital: $E'(A) < 0$. It follows that $\frac{\partial^2 e^i}{\partial A^i \partial x^i} = E'(A) < 0$. That is, more abatement capital decreases the amount of emissions, e^i, required per additional unit of output.
Furthermore, consider the output that can be produced with a given level of polluting input, $x^i(e^i, A^i) = \frac{e^i}{E(A^i)}$. The marginal productivity of abatement capital, $x^i_{A^i}(e^i, A^i) = \frac{-e^i E'(A^i)}{E(A^i)^2}$ is positive. It is assumed that $x^i_{A^i}$ is decreasing in A^i, so $x^i_{A^i A^i} < 0$.
Abatement capital can be accumulated by investment I^i at a cost of $C^i(I^i)$. Investment costs

are increasing and convex, $C^{i'} > 0$ and $C^{i''} > 0$. Abatement capital grows according to a standard capital accumulation function:

$$\dot{A}^i = I^i - \delta^i A^i \tag{2}$$

Here δ^i denotes a constant depreciation rate. Revenues are denoted by $R^i(x^i, x^j)$. The firm sells output x^i on the world market where it has to compete with firm j's output x^j. Let subscripts denote partial derivatives. It is assumed that $R^i_j < 0$, $R^i_{ij} < 0$, $R^i_{ii} < 0$ and $R^i_{jj} = 0$[5]. These assumptions imply that revenues decrease if competing output increases, and marginal revenues decrease in own and foreign output, while the marginal effect of foreign output does not change with foreign capital stocks. The firm's profits in the case of taxes read[6]:

$$\Pi^i = R^i(x^i, x^j) - (p^e + \tau^i)e^i - C^i(I^i) \tag{3}$$

From revenues, the firm must subtract the costs of the polluting input, e^i, and costs of investment in abatement, $C^i(I^i)$. In the case of emission standards, profits are similarly defined except that the firm does not have to pay emission taxes.

The marginal change in profits due to an additional unit of abatement capital will be called MBA^i, this is the derivative of Π^i to A^i, $\frac{\partial \Pi^i}{\partial A^i}$, with output at its equilibrium level. The following regularity condition is assumed to hold for all scenarios that are discussed in section 5:

$$\frac{\partial MBA^i}{\partial A^i}\frac{\partial MBA^j}{\partial A^j} > \frac{\partial MBA^i}{\partial A^j}\frac{\partial MBA^j}{\partial A^i} \tag{4}$$

This condition requires that own effects of abatement capital dominate cross effects. It ensures that open-loop equilibria exist.

The optimization problem faced by the firm can be summarized as follows. It has to decide on strategies for its output and investment to maximize its discounted stream of profits, given the capital accumulation function (2) and environmental policy. In case of taxes, this results in:

$$\max_{x^i \geq 0, I^i \geq 0} \int_0^\infty e^{-rt}[R^i(x^i, x^j) - (p^e + \tau^i)E(A^i)x^i - C^i(I^i)]dt \tag{5}$$

$$\text{s.t. } \dot{A}^i = I^i - \delta^i A^i \tag{6}$$

Here r is the rate of discount that is assumed to be equal for firms and governments. In case of standards, a similar optimization problem results, but then $\tau^i = 0$ and the following constraint on emissions is added:

$$e^i \leq \bar{e}^i \tag{7}$$

Whether environmental policy is binding depends on the world market price for e^i. If this

[5] e.g. $R^i = p(x^i, x^j)x^i$, with p a linear decreasing function of x^i and x^j has this characteristics.

[6] If environmental taxes are redistributed in a lump sum way, this increases the level of profits, but it will not change the decisions taken by firms.

is high enough, then in equilibrium firms would not emit more than the standard anyway. Here, it is assumed that prices are such that without government intervention, firms do not invest in emission reduction and the environmental standard is a binding constraint on their behaviour. A sufficient condition for zero investment in abatement without environmental policy is that the user costs of capital exceed the marginal private benefits of additional capital at $A^i = 0$. Let x^{iO} denote the Nash equilibrium output for $A^i = A^j = 0$. The marginal private benefits of additional capital consist of reductions in factor costs,$-p^e E'(0)x^{iO}$, and of a strategic advantage, $R^i_j x^j_{A^i}$, since foreign output is discouraged. This last term, which has a positive sign, is due to commitment effects. A larger capital stock provides firm i with a strategic advantage. This results in less output for firm j and hence higher revenues for firm i. The condition reads:

$$(r + \delta^i)C^{i\prime}(0) > -p^e E'(0)x^{iO} + R^i_j x^j_{A^i} \tag{8}$$

which assures that $A^i = A^j = 0$ is an equilibrium steady state. A formal proof is given in appendix A. Given this condition it is not optimal for a firm to invest in emission reduction, given that current levels of abatement capital are zero for both firms and no environmental policy is applied. It is assumed that condition (8) holds for all t for both firms and that at t=0 both firms have no abatement capital installed due to lack of environmental policy until that time.

The regulator has to set taxes or standards such that they balance firm objectives, Π^i, against environmental objectives as summarized by the damage function $D^i(P^i)$. The regulator's current welfare is then:

$$G^i = R^i(x^i, x^j) - p^e e^i - C^i(I^i) - D^i(P^i) \tag{9}$$

In case of taxes, its intertemporal decision problem is given by:

$$\max_{\tau^i} \int_0^\infty e^{-rt}[R^i(x^i, x^j) - p^e e^i - C^i(I^i) - D^i(P^i)]dt \tag{10}$$

$$\text{s.t. } I^i, x^i, I^j, x^j \text{ from firm behaviour}^7 \tag{11}$$

The government redistributes tax revenues in a lump sum way. In contrast to the firm it takes environmental damage into account. In the case of emission standards, the government optimizes its objective function with respect to the level of standards, $\bar{e}^i$.

Marginal social damage and marginal social benefits can now be defined. Both are defined per unit of emissions and for given levels of abatement capital. 'Social' here means from the macro economic point of view of the two countries together. Marginal social damage of one unit of polluting input used by firm i consists of marginal damage due to depositions $\beta^{ii}e^i$ in country i and marginal damage due to depositions $\beta^{ji}e^i$ in country j.

$$MD^i = D^{i\prime}\beta^{ii} + D^{j\prime}\beta^{ji} \tag{12}$$

[7] This is used as a shorthand for the equilibrium values of (5) to (6).

Marginal social (net) benefits of one unit of input used by firm i are determined by the amount of output, $1/E(A^i)$ that is produced with this unit. It consists of the marginal revenues to country i plus the (negative) marginal revenues to country j minus marginal production costs, i.e. the price p^e of one unit.

$$MB^i = \frac{1}{E(A^i)}[R^i_i + R^j_i] - p^e \tag{13}$$

From a static efficiency point of view, it is socially optimal if the government sets environmental policy such that the following condition holds:

$$MD^i = MB^i \tag{14}$$

This level of policy that equates marginal social damage to the marginal social benefits of emission is referred to as the 'Pigouvian' level of environmental policy. Similarly, the level of taxes that satisfies:

$$\tau^i = MD^i \tag{15}$$

will be referred to as the 'Pigouvian' level of taxes.

5 Equilibrium policy for three scenarios

5.1 The case of full cooperation

Consider the 'full cooperation' solution as a benchmark scenario. Assume that there is an overall government that wants the best for both countries together, while the two firms are owned by one overall manager. From the discussion in section 3 it follows that in this case there are no reasons to disturb policy from its Pigouvian level. The motives 2 and 4 disappear since there is no competition between firms. The remaining motives, 1 and 3 disappear since in the eyes of the overall government there is no transboundary pollution. Indeed it turns out that marginal social damage (MD^i) equals marginal social benefits (MB^i) in equilibrium.
The objective functions add the objectives of both firms and of both governments. In case of taxes, the 'cartel's' objective function is $\max_{x^i,I^i,x^j,I^j} \int_0^\infty e^{-rt}[\Pi^i + \Pi^j]dt$, while the overall government has the objective function $\max_{\tau^i,\tau^j} \int_0^\infty e^{-rt}[G^i + G^j]dt$, with Π^i as defined in equation (3) and G^i as defined in (9). For standards, obvious analogous objective functions can be formulated. The cooperative firms' manager divides production and investment in an optimal way over firm i and firm j, to maximize total profits. It takes environmental policy as given. The cooperative governments are a Stackelberg leader and set policy in country i and j to maximize welfare.
Analysis of feedback Stackelberg equilibrium conditions (see appendix B) shows that in equilibrium (14) must hold for both emission taxes and emission standards. It is understandable that it is optimal to set policy at its Pigouvian level since transboundary pollution nor trade strategic

motives play a role here. Hence none of the reasons to disturb policy from its Pigouvian level are present.

The first-order conditions for an open-loop Stackelberg equilibrium are also given in appendix B. The cooperative firms' manager chooses an optimal time path of investment, given a time path of policy levels. He balances adjustment costs against marginal revenues of additional abatement capital (reduced future payments for the use of e^i). The government takes firm behaviour as given and sets an optimal path of policy. In case of taxes it can be shown (see appendix B) that a time-consistent policy path satisfies the first-order conditions and is an optimal path. This path is characterized by (14) or equivalently by (15). In case of standards on emissions, manipulation of the first-order conditions for an open-loop Stackelberg equilibrium leads similarly to the condition (14). It can be proved that the solution is time consistent (see appendix B)).

It follows that in an open-loop Stackelberg equilibrium with one overall government and one overall manager the government chooses the Pigouvian level of environmental policy. Moreover the time paths of the variables in open-loop equilibrium equals the time paths in feedback equilibrium. The latter follows, since setting environmental policy at the Pigouvian level, enables the government to reach the socially optimal path of investment in environmental improvement and of output in both equilibria. In case of open-loop strategies, there is no time inconsistency, because the government achieves the social optimum. It does not need to try to improve on this result at a later time.

5.2 Cooperative government with competition between firms

As a second scenario, the case where cooperating governments regulate two firms that compete with each other is considered. The relevant objective functions are then $\max_{x^i, I^i} \int_0^\infty e^{-rt}[\Pi^i]dt$, and $\max_{\tau^i, \tau^j} \int_0^\infty e^{-rt}[G^i + G^j]dt$, with Π^i as defined in equation (3) and G^i as defined in (9) in case of taxes. In case of standards, obvious analogues can be formulated. Since firms compete with each other, while governments cooperate, some of the effects mentioned in section 3 work in a different direction in this scenario. For example firms derive no utility from increases in profits of the foreign firm when they decrease their own output, while the common government values both firms' profits. Compared to the social optimum, firms produce too much output. Therefore effect 4 now induces the governments to set stricter policies to reduce output.

In case of a feedback Stackelberg equilibrium it can be proved (see appendix B) that the condition (14) must be satisfied by an optimal environmental policy. Governments cooperate, so they still equate marginal social benefits to marginal social damage, like in the first case. In case of taxes, from the first-order conditions on a firm's choice of output, that is, $R_i^i = (p^e + \tau^i)E(A^i)$, from condition (14) and definition (12) it follows that: $\tau^i = MD^i - \frac{R_i^j}{E(A^i)}$. The upward distortion ($-R_i^j > 0$) of taxes compared to the scenario in section 5.1 is due to effect 4 as explained above. Note that, since R_i^j and MD^i depend on x^i, which in turn depends on τ^i, the righthand-side of the expression above still contains τ^i. But this is a convenient way to represent the equilibrium conditions, since the expression clarifies what distortions are present.

In case of standards, (14) is an equilibrium condition.
For an open-loop Stackelberg equilibrium the first-order condition for optimal policy is:

$$MB^i = MD^i + \nu^i \frac{B^i}{E(A^i)} + \nu^j \frac{B^j}{E(A^j)} \tag{16}$$

under taxes. The factors B^i and B^j are complex expressions (see appendix B) that denote the combined effect of marginal policy changes on the two firms' valuation of abatement capital. Under standards, the condition is:

$$MB^i = MD^i + \nu^i MBA^{iN}_{\bar{e}^i} + \nu^j MBA^{jN}_{\bar{e}^i} \tag{17}$$

The ν^i denote the shadowprice of the cooperative government for the valuation by firm i (i=1,2) of its own abatement capital. These terms signal the time inconsistency that exists in the open-loop equilibrium in this case where firms compete. A rough explanation for the existence of time inconsistency goes as follows. Due to firms' competition, the cooperative governments can at most reach a second-best solution. They apply open-loop strategies with commitment. Hence at a certain point in time, they decide on an optimal policy path for the whole future. At any given future point in time t however, governments could steer firms to a better solution for the remaining future. For, at such a point, abatement capital stocks are given to the firms and can only be adjusted gradually through investment. Governments would prefer to change their policy once time t is reached. But they have committed themselves to the chosen policy path. As noticed by Kennedy (1994), environmental policy can at most be a second-best instrument for the scenario with competition between firms, since there are two goals to be attained by one instrument (reduction of strategic behaviour, which is welfare decreasing in this case, and reduction of environmental damage). So the first-best optimum is not obtainable. The time inconsistency is a sign that the feedback solution, which by definition is time consistent, is characterized by a different time path of investment in abatement capital and environmental policy than the open-loop solution[8].
If firms would be in perfect competition, they would not be able to influence each other's revenues through output. Then taxes in a feedback equilibrium are not distorted from their Pigouvian level. There is no reason for cooperating governments to distort them, for there is no oligopolistic competition among firms which they can change. In open-loop equilibrium the distortions are absent as well, for the same reason that the government cannot influence competition if it is perfect. That can be seen from a consideration of the B^i-factors in appendix B. These factors equal zero in case of perfect competition. If standards are applied as the environmental policy instrument, and firms are in perfect competition, the feedback equilibrium condition is (14) as before. However, in open-loop equilibrium, not all distortions disappear. Under standards, the government has a larger influence on firms output choices than under taxes. That implies that in an open-loop equilibrium, time inconsistency is still present, even

[8]Therefore, considering a full dynamic model shows that the predictions of multistage models, whose equilibria are comparable to our open-loop equilibrium, may be wrong if the players' interaction is better described by a feedback equilibrium.

if firms are in perfect competition. The term $MBA^i_{\bar{e}^i}$ namely does not disappear completely.

5.3 Competition between firms and between governments

In a situation of strategic international trade, when both governments and firms compete with each other, all the reasons for policy distortion mentioned in section 3 are present and work the way they were presented there. The objective functions are given by $\max_{x^i, I^i} \int_0^\infty e^{-rt}[\Pi^i]dt$, and $\max_{\tau^i} \int_0^\infty e^{-rt}[G^i]dt$, with Π^i as defined in equation (3) and G^i as defined in (9), in case of taxes. In case of standards, governments optimize over standards and firms' objective functions are adjusted accordingly. The feedback Stackelberg equilibrium with emission standards can be summarized by (see appendix B):

$$MB^i = MD^i - D^{j\prime}\beta^{ji} + R^j_i \frac{1}{E(A^i)} \tag{18}$$

Standards deviate from the Pigouvian level due to the distortionary effects 1 and 4. Governments neglect foreign damage $D^{j\prime}\beta^{ji}$ and set standards which are too lax, so that marginal damage exceeds marginal benefits. Furthermore, effect 4 leads governments to stimulate production by their own firms. This is represented by the term $\frac{R^j_i}{E(A^i)}$ in the formula above.
In case of emission taxes, the condition for optimal policy is (see appendix B):

$$MB^i = MD^i - D^{j\prime}\beta^{ji} + R^j_i \frac{1}{E(A^i)} + D^{i\prime}\beta^{ij}\frac{E(A^j)}{E(A^i)}\frac{x^{Jn}_{\tau^i}}{x^{iN}_{\tau^i}} - R^i_j \frac{1}{E(A^i)}\frac{x^{jN}_{\tau^i}}{x^{iN}_{\tau^i}} \tag{19}$$

Compared to the case of standards, two more distortionary effects (2 and 3) are present, since firms have indirect influence on each others output through the market price. They discourage foreign production to reduce damage from abroad (effect 3, represented by the term: $D^{i\prime}\beta^{ij}\frac{E(A^j)}{E(A^i)}\frac{x^{jN}_{\tau^i}}{x^{iN}_{\tau^i}}$). And they discourage foreign production out of a rent shifting motive (effect 2, represented by the term: $-\frac{R^i_j}{E(A^i)}\frac{x^{jN}_{\tau^i}}{x^{iN}_{\tau^i}}$). Both terms are negative and lead to too lax policies. Using the condition on equilibrium output choice, $R^i_i = (p^e + \tau^i)E(A^i)$ and definition (12) the condition on optimal taxes can be rewritten:

$$\tau^i = MD^i - D^{j\prime}\beta^{ji} + D^{i\prime}\beta^{ij}\frac{E(A^j)}{E(A^i)}\frac{x^{jN}_{\tau^i}}{x^{iN}_{\tau^i}} - R^i_j \frac{1}{E(A^i)}\frac{x^{jN}_{\tau^i}}{x^{iN}_{\tau^i}} \tag{20}$$

Both in case of taxes and in case of standards, environmental policies are distorted and set too lax from a social point of view.
The open-loop Stackelberg equilibrium is characterized by the following condition on environmental policy in case of emission standards:

$$MB^i = MD^i - D^{j\prime}\beta^{ji} + R^j_i\frac{1}{E(A^i)} - \nu^{ii}\frac{E'(A^i)}{E(A^i)}[R^i_i\frac{1}{E(A^i)} + R^i_{ii}\frac{\bar{e}^i}{E(A^i)^2}] - \nu^{ij}R^j_{ij}\frac{\bar{e}^j}{E(A^i)}\frac{E'(A^j)}{E(A^j)^2} \tag{21}$$

In case of taxes the condition is:

$$MB^i = MD^i - D^{j\prime}\beta^{ji} + R^j_i \frac{1}{E(A^i)} + D^{i\prime}\beta^{ij}\frac{E(A^j)}{E(A^i)}\frac{x^{jN}_{\tau^i}}{x^{iN}_{\tau^i}} - R^i_j \frac{1}{E(A^i)}\frac{x^{jN}_{\tau^i}}{x^{iN}_{\tau^i}} +$$

$$\frac{1}{E(A^i)}[\nu^{ii}\frac{1}{x^{iN}_{\tau^i}}\frac{\partial MBA^i}{\partial \tau^i} + \nu^{ij}\frac{1}{x^{iN}_{\tau^i}}\frac{\partial MBA^j}{\partial \tau^i}] \tag{22}$$

Both conditions are derived in appendix B. Like in the second benchmark case, the open-loop equilibrium solutions are not time consistent. This can be seen from the presence of the nonzero terms with ν^{ii} and ν^{ij} representing the government's valuation of changes in the shadowprice of capital as perceived by its home firm (ν^{ii}) and the foreign firm (ν^{ij}), respectively.

In general the direction of distortions in the case of open-loop strategies (and in case of comparable multistage models) is inconclusive. This is due to the twofold effect of an increase in abatement capital. More abatement capital leads to less emissions per unit of output. But, more abatement capital may also lead to increased output and hence emissions, because production costs per unit are lowered. For governments there is no general rule how and in what direction to steer abatement investments. A stricter tax policy, for instance, increases the profitability of investment in emission reductions for each unit of output produced. But it also reduces the optimal level of output and hence decreases the profitability of a given investment in emission reduction. Hence it is inconclusive whether policies are distorted upward or downward. For feedback investment strategies that is not the case. Firms base their investment on their expectations about future policies. When current policies imply no commitments to future levels of environmental policy, as is the case in the feedback equilibrium, governments cannot directly influence investment in abatement technology with current policies. In this case the only distortions in policy levels are due to transboundary pollution and strategic trade effects.

Compare the results above to a situation with only transboundary pollution. That can be done by considering (18) and (20) with $R^i_j = R^j_i = 0$. When firms do not try to influence each other's output, the terms with these derivatives and therefore also $x^i_{\tau^i}$ and $x^j_{\tau^i}$ disappear. Condition (18) then becomes:

$$MB^i = MD^i - D^{i\prime}\beta^{ji} \tag{23}$$

while condition (20) becomes:

$$\tau^i = MD^i - D^{j\prime}\beta^{ji} \tag{24}$$

The distortions numbered 2 and 3 disappear, since they are due to strategic interaction among firms. The resulting policies are closer to their Pigouvian levels. As a result output as well as environmental damage is lower than in the equilibrium under the scenario described in section 5.3.

6 Summary and conclusions

This chapter describes environmental policy setting by national governments that balance environmental targets with domestic firm competitiveness in a dynamic game model of an international duopoly. Three different scenarios were considered for two types of decision strategies on investment and environmental policy.

In the benchmark case, scenario 5.1, where governments and firms cooperate, no distortions from the Pigouvian policy occur and governments are able to reach the first-best solution. In this specific case open-loop and feedback strategies result in the same equilibrium.

If firms compete, while governments cooperate, scenario 5.2 applies. Then, governments have the double target to reduce environmental damage as well as competition among firms. Only a second best solution can be reached with distortions in the environmental policy instruments. These distortions are the same with and without transboundary pollution, since the cooperating governments consider damage in both countries. Under perfect competition there would be no distortions at all. With oligopolistic firms, distortions exist and open-loop strategies are not time consistent. It is undetermined whether policy instruments are distorted upward or downward. Feedback strategies are time consistent and it can be seen that these are distorted downward.

The third scenario with competition between firms and between governments is characterized by distortions as well. Now, governments want to provide their domestic firms with a competitive advantage. Furthermore, each government only considers domestic damage. In open-loop equilibrium, the sign of the distortions is undetermined and the equilibrium strategies are not time consistent. Feedback equilibrium strategies are time consistent. These strategies are always distorted downwards, that is, environmental policy is laxer than the Pigouvian level.

With trade that is characterized by strategic interaction there are several additional reasons, compared to trade under perfect competition, for regulators to deviate from the Pigouvian level of environmental policy. Apart from the usual distortions due to transboundary pollution, governments may also distort policies to increase a domestic firm's profits.

Compared to a multistage model, a full dynamic model has the advantage that additional possible means of interaction between firms and governments can be analysed. With the most rigid interaction considered in this chapter, an equilibrium where governments use open-loop strategies and firms use open-loop investment strategies, results are in general inconclusive and time inconsistent. If feedback investment and government strategies are considered, though, the sign of the distortion of environmental policy can be characterized.

The formulas above characterize policy setting along equilibrium time-paths. The analysis in chapter 7 uses this to investigate the investment paths of firms in the model more carefully.

In this model, with perfect knowledge and no uncertainty it seems logic that standards and taxes are equivalent instruments of environmental policy. However, consider the feedback equilibrium. As mentioned, the signs of the distortions in case of feedback strategies can be determined. Both under taxes and under standards, environmental policy is set at laxer than socially optimal levels. But in case of emission standards, firms' commitment to a certain output level is greater, because the standard is a binding constraint on output for given abatement capital stocks. Therefore, it is more difficult for the government to affect the output choices of the competitor with standards than with taxes. Some distortionary effects are absent under standards. It cannot be concluded however, that standards are always less distorted, because the remaining distortions may be stronger under standards. But it is clear that equilibrium output and emission levels differ under the two instruments, even in this model with complete knowledge and no uncertainty.

With feedback strategies, current policies do not influence the investment behaviour of firms since the latter is based on the future profitability of investments. This conclusion at the same time clarifies the circumstances under which the result holds. The analysis assumes perfect foresight so that firms are indeed able to base their investment decisions on expectations about the future. In reality such foresight is absent and it may very well be the case that current taxes are used to estimate future taxes. To compare what happens when firms are myopic and base their decisions on current tax levels is a possibility for further research. In a closed economy framework, models with myopic firms subject to environmental policy are analysed in Magat (1978).

An extension of the analysis presented here would be the inclusion of consumer surplus, as in Kennedy (1994) or Ulph (1996a) so that two more aspects of distorted environmental policy, the increase in polluting production in foreign countries and the benefits to consumers from increased production, could be analysed. The result of the inclusion of consumer surplus depends on how large a part of total production is consumed at home. If home consumers are only a small fraction of total demand, the results are not expected to change very much.

A The case of no environmental policy

In this appendix it is proved that in case no environmental policy is applied, the equilibrium steady state is characterized by $A^i = A^j = 0$, when condition (8) is satisfied, both for open-loop and feedback strategies. The proof is only given for the case of Nash competition between firms. For the case of cooperation between the two firms, very similar derivations result in slightly different conditions. This proof is omitted.

A firm not subject to environmental policy and in Nash competition with the other firm has to solve the optimization problem defined by (5) to (6) with the tax rate set at zero. Because its competitor solves the same problem, a differential game results. Consider first the equilibrium conditions for output. Since output has no effect on the dynamics, the equilibrium condition is just the standard condition that marginal benefits equal marginal cost (where R^i_i denotes $\frac{\partial R^i}{\partial x^i}$). In case of an interior solution, this is:

$$R^i_i = p^e E(A^i) \tag{A.1}$$

If it is furthermore assumed that the firm uses a feedback strategy for its investment choice, the following

Hamilton-Jacobi-Bellman condition, where equilibrium output has been inserted, must hold for i=1,2:

$$rU^i(A^i, A^j) = \max_{I^i}[R^i - p^e E(A^i)x^i - C(I^i) + U^i_{A^i}(I^i - \delta^i A^i) + U^i_{A^j}(I^j - \delta^i A^j)], \qquad (A.2)$$

Necessary and sufficient conditions on equilibrium investment, $\hat{I}$, are then:

$$C^{i\prime}(\hat{I}^i) \geq U^i_{A^i}; \;\; \hat{I}^i \geq 0; \;\; \hat{I}^i[U^i_{A^i} - C^{i\prime}(\hat{I}^i)] = 0 \qquad (A.3)$$

It is assumed that the value functions U^i are differentiable. From (A.3) it follows that $\hat{I}^i = 0$ is the equilibrium investment strategy for $A^i = A^j = 0$, if

$$U^i_{A^i}(0,0) < C'(0) \qquad (A.4)$$

Moreover, since the inequality is strict, $\hat{I}^i_{A^i} = \hat{I}^i_{A^j} = 0$. To find an expression for $U^i_{A^i}(0,0)$, take the full derivatives of the Hamilton-Jacobi-Bellman conditions to A^i and A^j and evaluate them in $(0,0)$. After rearrangement and the insertion of equilibrium output and equilibrium investment the following expression results:

$$(r + \delta^i)U^i_{A^i}(0,0) = -p^e E'(0)x^{iN}(0,0) + R^i_{x^j} x^{jN}_{A^i}(0,0) \qquad (A.5)$$

If this is inserted in (A.4) then (8) results, where x^{iO} denotes $x^{iN}(0,0)$:

$$C'(0) > \frac{1}{r + \delta^i}[-p^e E'(0)x^{iO} + R^i_j x^{jO}_{A^i}] \qquad (A.6)$$

If this condition is satisfied for both firms, $\hat{I}^i = 0$ is the equilibrium choice of investment, so that $A^i = A^j = 0$ is an equilibrium steady state.

For open-loop strategies, the first-order conditions for equilibrium investment in the no policy case are:

$$C'(I^i) \geq \lambda^i; \;\; I^i \geq 0; \;\; I^i[C'(I^i) - \lambda^i] = 0 \qquad (A.7)$$

$$\dot{\lambda}^i = (r + \delta^i)\lambda^i - MBA^i \qquad (A.8)$$

with $MBA^i = -p^e x^{iN} E'(A^i) + R^i_j x^{jN}_{A^i}$. If the condition (8) holds, then $\hat{I}^i = 0$ and $\lambda^i = \frac{MBA^i}{r+\delta^i}$ satisfy conditions (A.7) and (A.8) for $A^i = A^j = 0$. This is necessary condition for $A^i = A^j = 0$ to be an equilibrium steady state. If appropriate conditions are satisfied, it is also sufficient. That requires at least that $MBA^i_{A^i} \leq 0$ in an environment of $A^i = A^j = 0$. Here MBA^i denotes the marginal benefits per time unit from a unit of abatement capital. These are reductions in payments for the polluting input, $p^e x^{iN} E'(A^i)$, and a strategic term, $R^i_j x^{jN}_{A^i}$, due to the competitive advantage gained by a more efficient production process. This second term gives the marginal decrease in competing output and the effect this has on own firm revenues.

B Details for section 5

Appendix B provides the details for section 5. Each scenario is considered in a separate section. Each section gives first the conditions for equilibrium output, then details on the feedback equilibrium and on the open-loop equilibrium. In this appendix, subscripts denote partial derivatives. A variable y in equilibrium is represented by $\hat{y}$. Partial derivatives of revenues to output, $\frac{\partial R^i}{\partial x^i}$ and $\frac{\partial R^i}{\partial x^j}$, are denoted by R^i_i and R^i_j. Furthermore, $\mathbf{A}$, τ, $\mathbf{x}$, and $\bar{\mathbf{e}}$ denote the vectors (A^i, A^j), (τ^i, τ^j), (x^i, x^j) and $(\bar{e}^i, \bar{e}^j)$ respectively. Remember that marginal benefits per unit of emission, MB^i, were defined by:

$$MB^i = \frac{1}{E(A^i)}(R^i_i + R^j_i) - p^e \tag{B.1}$$

and that marginal damage was defined by:

$$MD^i = D^{i\prime}\beta^{ii} + D^{j\prime}\beta^{ji} \tag{B.2}$$

Current welfare in country i was defined in (9) by:

$$G^i = R^i - p^e e^i - C^i - D^i \tag{B.3}$$

These definitions are used repeatedly in the rest of the appendix. Recall the definition (1) of pollution in country i, P^i. The derivative of P^i, to a tax τ^j is:

$$P^i_{\tau^j} = \beta^{ii} E(A^i) x^i_{\tau^j} + \beta^{ij} E(A^j) x^j_{\tau^j} \tag{B.4}$$

The derivative of pollution in country i to a standard $\bar{e}^j$ is:

$$P^i_{\bar{e}^j} = \beta^{ij} \tag{B.5}$$

Scenario 1: Full cooperation

The first-order conditions for optimal choice of output for an overall firm manager in country i are in case of taxes:

$$R^1_i + R^2_i \leq (p^e + \tau^i) E(A^i) \tag{B.6}$$

$$x^i \geq 0 \tag{B.7}$$

$$x^i [R^1_i + r^2_i - (p^e + \tau^i) E(A^i)] = 0 \tag{B.8}$$

All three conditions must hold for i=1,2. Nash equilibrium results in output choices $x^{1C}(\mathbf{A}, \tau)$ and $x^{2C}(\mathbf{A}, \tau)$. Assume that an interior solution with both outputs positive is reached. Then (B.6) holds with equality. If these optimal output choices are inserted, the following differential game results:

manager:

$$\max_{I^1 \geq 0, I^2 \geq 0} \int_0^\infty e^{-rt} \left[\Pi^1 + \Pi^2\right] dt$$

$$\text{s.t. } \Pi^i = R^i(\mathbf{x}^{\mathbf{C}}) - (p^e + \tau^i)E(A^i)x^{iC} - C^i(I^i)$$

$$\dot{A}^i = I^i - \delta^i A^i$$

government:

$$\max_{\tau^1, \tau^2} \int_0^\infty e^{-rt} \left[G^1 + G^2\right] dt$$

s.t. firm behaviour

The phrase 'firm behaviour' is used as a shorthand for the equilibrium values of $\hat{I}^i$, $\hat{I}^j$, $\hat{x}^i$ and $\hat{x}^j$ that follow from solution of the firms' problem for given taxes. That is, the government, which is the Stackelberg leader, takes into account the equilibrium behaviour of the cartel's manager.

In case of an emissions standard, $\bar{e}^i$, for the overall firm manager, the first-order conditions for optimal output choice read:

$$R_i^1 + R_i^2 \leq (p^e + \phi^i)E(A^i) \tag{B.9}$$

$$x^i \geq 0 \tag{B.10}$$

$$x^i[R_i^1 + R_i^2 - (p^e + \phi^i)E(A^i)] = 0 \tag{B.11}$$

$$\phi^i \geq 0 \tag{B.12}$$

$$E(A^i)x^i \leq \bar{e}^i \tag{B.13}$$

$$\phi^i[\bar{e}^i - E(A^i)x^i] = 0 \tag{B.14}$$

for i=1,2. Here $\phi^i + p^e$ is the shadow price for the polluting input. If $R_i^1(\mathbf{x}) + R_i^2(\mathbf{x}) \geq p^e E(A^i)$ for $x^i = \frac{\bar{e}^i}{E(A^i)}$, the standard is binding for a given output x^j. Assume that, for $i = 1, 2$, it holds:

$$R_i^1\left(\frac{\bar{e}^1}{E(A^1)}, \frac{\bar{e}^2}{E(A^2)}\right) + R_i^2\left(\frac{\bar{e}^1}{E(A^1)}, \frac{\bar{e}^2}{E(A^2)}\right) \geq p^e E(A^i) \tag{B.15}$$

Then the case where both standards are binding and $x^{iC} = \frac{\bar{e}^i}{E(A^i)}$, for i=1,2, is an equilibrium. The differential game reduces to:

manager:

$$\max_{I^1 \geq 0, I^2 \geq 0} \int_0^\infty e^{-rt} \left[\Pi^1 + \Pi^2\right] dt$$

s.t. $\Pi^i = R^i - p^e \bar{e}^i - C^i(I^i)$

$$\dot{A^i} = I^i - \delta^i A^i$$

government:

$$\max_{\bar{e}^1, \bar{e}^2} \int_0^\infty e^{-rt} \left[G^1 + G^2\right] dt$$

s.t. firm behaviour

Feedback investment

Under full cooperation, the Hamilton-Jacobi-Bellman conditions are in case of taxes:

$$rU(\mathbf{A}) = R^1 + R^2 - (p^e + \hat{\tau}^1)E(A^1)x^{1C} - (p^e + \hat{\tau}^2)E(A^2)x^{2C} - C^1 - C^2 + U_{A^1}(\hat{I}^1 - \delta^i A^1) + U_{A^2}(\hat{I}^2 - \delta^i A^2) \quad \text{(B.16)}$$

$$rW(\mathbf{A}) = R^1 + R^2 - p^e E(A^1)x^{1C} - p^e E(A^2)x^{2C} - C^1 - C^2 - D^1 - D^2 + W_{A^1}(\hat{I}^1 - \delta^i A^1) + W_{A^2}(\hat{I}^2 - \delta^i A^2) \quad \text{(B.17)}$$

Here U and W are the value functions of the cooperating firms, respectively the cooperating governments. $\hat{I}^i$ is the equilibrium rate of investment in abatement and $\hat{\tau}^i$ the equilibrium rate of taxes. The equilibrium rate of investment must satisfy the following first-order conditions:

$$C^{i\prime}(\hat{I}^i) \geq U_{A^i}(\mathbf{A}); \;\; \hat{I}^i \geq 0; \;\; \hat{I}^i[C^{i\prime}(\hat{I}^i) - U_{A^i}] = 0 \quad \text{(B.18)}$$

These result in $\hat{I}^i(\mathbf{A})$, so the firm's rate of investment is not directly influenced by environmental taxes. The equilibrium rate of taxes, $\hat{\tau}^i$, must satisfy the following first-order conditions:

$$\tau^1 E(A^1)x^{1C}_{\tau^1} + \tau^2 E(A^2)x^{2C}_{\tau^1} = D^{1\prime} P^1_{\tau^1} + D^{2\prime} P^2_{\tau^1} \quad \text{(B.19)}$$

$$\tau^1 E(A^1)x^{1C}_{\tau^2} + \tau^2 E(A^2)x^{2C}_{\tau^2} = D^{1\prime} P^1_{\tau^2} + D^{2\prime} P^2_{\tau^2} \quad \text{(B.20)}$$

These follow if the value function W is maximized over τ^i and (B.6) is inserted. Use (B.4) to rewrite (B.19) and (B.20) to:

$$E(A^1)x^{1C}_{\tau^i}[\tau^1 - D^{1\prime}\beta^{11} - D^{2\prime}\beta^{21}] = -E(A^2)x^{2C}_{\tau^i}[\tau^2 - D^{1\prime}\beta^{12} - D^{2\prime}\beta^{22}] \quad \text{(B.21)}$$

for i=1,2. It follows that

$$\hat{\tau}^i = D^{i'}\beta^{ii} + D^{j'}\beta^{ji} \tag{B.22}$$

or, inserting definition (B.2),

$$\hat{\tau}^i = MD^i \tag{B.23}$$

Use the first-order condition on output, (B.6), equation (B.23) and definition (B.1) to find that in equilibrium:

$$MB^i = MD^i, \tag{B.24}$$

as it is stated in the main text. In case of emission standards, the Hamilton-Jacobi-Bellman equations read:

$$rU(\mathbf{A}) = R^1 + R^2 - p^e\hat{\bar{e}}^1 - p^e\hat{\bar{e}}^2 - C^1(\hat{I}^1) - C^2(\hat{I}^2) + U_{A^1}(\hat{I}^1 - \delta^i A^1) + U_{A^2}(\hat{I}^2 - \delta^i A^2) \tag{B.25}$$

$$rW(\mathbf{A}) = R^1+R^2-p^e\hat{\bar{e}}^1-p^e\hat{\bar{e}}^2-C^1(\hat{I}^1)-C^2(\hat{I}^2)-D^1-D^2+W_{A^1}(\hat{I}^1-\delta^i A^1)+W_{A^2}(\hat{I}^2-\delta^i A^2) \tag{B.26}$$

Conditions for the equilibrium rate of investment are given in (B.18). The equilibrium rates of standards, $\hat{\bar{e}}^i$, follow from the first-order conditions for the maximization of W over $\bar{e}^i$:

$$(R_i^1 + R_i^2)\frac{1}{E(A^i)} - p^e - D^{1'}P^1_{\bar{e}^1} - D^{2'}P^2_{\bar{e}^i} = 0 \tag{B.27}$$

for i=1,2. Use (B.2), (B.1) and (B.5) to rewrite this to:

$$MB^i = MD^i, \tag{B.28}$$

as it is stated in the main text.

Open-loop investment

In case of an open-loop Stackelberg equilibrium, the first-order conditions on investment for cooperating firms read:

$$\dot{A}^i = I^i - \delta^i A^i \tag{B.29}$$

$$C^{i'}(I^i) \geq \lambda^i;\ \ I^i \geq 0;\ \ I^i[C^{i'}(I^i) - \lambda^i] = 0 \tag{B.30}$$

$$\dot{\lambda}^i = (r + \delta^i)\lambda^i - MBA^{iC} \tag{B.31}$$

for i=1,2. Here MBA^{iC} denotes the marginal benefits per time unit of a unit of abatement capital in firm i for the firms, in case of cooperation. In case of taxes, $MBA^{iC} = -(p^e + \tau^i)x^{iC}E'(A^i)$. In case of emission standards, $MBA^{iC} = -[R_i^1 + R_i^2]\frac{\bar{e}^i E'(A^i)}{E(A^i)^2}$.
The first-order conditions to the government read in case of taxes:

$$[R_1^1 + R_1^2 - p^e E(A^1)]x_{\tau^i}^{1C} + [R_2^1 + R_2^2 - p^e E(A^2)]x_{\tau^i}^{2C} -$$

$$D^{1'}P_{\tau^i}^1 - D^{2'}P_{\tau^i}^2 - \nu^1\frac{\partial MBA^{1C}}{\partial \tau^i} - \nu^2\frac{\partial MBA^{2C}}{\partial \tau^i} = 0 \tag{B.32}$$

$$\dot{\mu}^i = (r + \delta^i)\mu^i - [R_1^1 + R_1^2 - p^e E(A^1)]x_{A^i}^{1C} - [R_2^1 + R_2^2 - p^e E(A^2)]x_{A^i}^{2C} +$$

$$p^e E'(A^i)x^{iC} + D^{1'}P_{A^i}^1 + D^{2'}P_{A^i}^2 + \nu^1\frac{\partial MBA^{1C}}{\partial A^i} + \nu^2\frac{\partial MBA^{2C}}{\partial A^i} \tag{B.33}$$

$$\dot{\nu}^i = -\delta^i\nu^i + \frac{C^{i'}(I^i) - \mu^i}{C^{i''}(I^i)} \tag{B.34}$$

$$\nu^i(0) = 0 \tag{B.35}$$

for i=1,2. Here μ^i is the shadow value for abatement capital in firm i and ν^i is the shadow value that the government as a Stackelberg leader attaches to λ^i. Insert the first-order condition on output, (B.6), in (B.32) and (B.33) and try the solution $\nu^1(t) = \nu^2(t) = 0$. For this solution, all conditions in (B.29) to (B.35) are satisfied if taxes are set according to:

$$\tau^i = D^{i'}\beta^{ii} + D^{j'}\beta^{ji} \tag{B.36}$$

for i=1,2, for all t.
In case of emission standards, the open-loop first-order conditions for cooperating governments read:

$$[R_i^1 + R_i^2]\frac{1}{E(A^i)} - p^e - D^{1'}\beta^{1i} - D^{2'}\beta^{2i} - \nu^1\frac{\partial MBA^{1C}}{\partial \bar{e}^i} - \nu^2\frac{\partial MBA^{2C}}{\partial \bar{e}^i} = 0 \tag{B.37}$$

$$\dot{\mu}^i = (r + \delta^i)\mu^i - [R_i^1 + R_i^2]x_{A^i}^{iC} + \nu^1\frac{\partial MBA^{1C}}{\partial A^i} + \nu^2\frac{\partial MBA^{2C}}{\partial A^i} \tag{B.38}$$

$$\dot{\nu}^i = -\delta^i\nu^i + \frac{(C^{i'} - \mu^i)}{C^{i''}} \tag{B.39}$$

$$\nu^i(0) = 0 \tag{B.40}$$

for i=1,2. The first-order conditions for cooperating firms were given in (B.29) to (B.31). Use $x^i = \frac{\bar{e}^i}{E(A^i)}$ and try the solution $\nu^1(t) = \nu^2(t) = 0$. It follows that all conditions in (B.29) to (B.31) and (B.37) to

(B.40) are satisfied if standards are set such that:

$$(R_i^1 + R_i^2)\frac{1}{E(A^i)} - p^e = D^{1'}\beta^{1i} + D^{2'}\beta^{2i} \tag{B.41}$$

or, equivalently,

$$MB^i = MD^i \tag{B.42}$$

The latter condition follows immediately from (B.2) and (B.1).

So for both instruments, a time-consistent open-loop equilibrium exists for this scenario. The equilibrium is, moreover, the first-best solution. To see this, consider the cooperative social-planning problem with objective function:

$$\max_{I^1, I^2, x^1, x^2} \int_0^\infty e^{-rt}[R^1 + R^2 - p^e(e^1 + e^2) - C^1(I^1) - C^2(I^2) - D^1(P^1) - D^2(P^2)]dt \tag{B.43}$$

The first-order conditions for investment and output for the social planning problem equal (B.29) to (B.31) and (B.6) respectively (B.9) to (B.14), with environmental policy (B.36) or (B.41) inserted. With these policies, the cooperating governments therefore reach the first-best solution. Furthermore, the optimality conditions are the same for feedback and open-loop strategies.

Scenario 2: Competition between firms and cooperating governments

Assume that in case of taxes an interior equilibrium results. Then the Nash equilibrium conditions for output are in case of taxes:

$$R_i^i = (p^e + \tau^i)E(A^i) \tag{B.44}$$

for i=1,2. The Nash equilibrium output choices are denoted $x^{iN}(\mathbf{A}, \tau)$. If these output choices are inserted, the following differential game results:

manager of firm i:

$$\max_{I^i \geq 0} \int_0^\infty e^{-rt}\,[\Pi^i]\,dt$$

$$\text{s.t. } \Pi^i = R^i(\mathbf{x^N}) - (p^e + \tau^i)E(A^i)x^{iN} - C^i(I^i)$$

$$\dot{A}^i = I^i - \delta^i A^i$$

cooperating governments:

$$\max_{\tau^1, \tau^2} \int_0^\infty e^{-rt}\,[G^1 + G^2]\,dt$$

s.t. firm behaviour

Assume that both standards are binding in equilibrium. Then in case of standards $x^{iN} = \frac{\bar{e}^i}{E(A^i)}$. The assumption

$$R^i_i(\frac{\bar{e}^1}{E(A^1)}, \frac{\bar{e}^2}{E(A^2)}) \geq p^e E(A^i) \tag{B.45}$$

for i=1,2, ensures that this is a Nash equilibrium indeed. The differential game reduces to:
manager of firm i:

$$\max_{I^i \geq 0} \int_0^\infty e^{-rt} \, [\Pi^i] \, dt$$

s.t. $\Pi^i = R^i(\mathbf{x^N}) - p^e \bar{e}^i - C^i(I^i)$ i=1,2

$\dot{A}^i = I^i - \delta^i A^i$ i=1,2

cooperating governments:

$$\max_{\bar{e}^1, \bar{e}^2} \int_0^\infty e^{-rt} \, [G^1 + G^2] \, dt$$

s.t. firm behaviour

Feedback investment

The Hamilton-Jacobi-Bellman equations for scenario 2 are in case of taxes:

$$rU^i(\mathbf{A}) = R^i - (p^e + \hat{\tau}^i)E(A^i)x^{iN} - C^i(\hat{I}^i) + U^i_{A^i}(\hat{I}^i - \delta^i A^i) + U^i_{A^j}(\hat{I}^j - \delta^i A^j) \tag{B.46}$$

$$rW(\mathbf{A}) = R^1 + R^2 - p^e E(A^1)x^{1N} - p^e E(A^2)x^{2N} - C^1 - C^2 - D^1 - D^2 + W_{A^1}(\hat{I}^1 - \delta^i A^1) + W_{A^2}(\hat{I}^2 - \delta^i A^2) \tag{B.47}$$

for i=1,2. Here U^i is the value function of firm i and W is value function for the cooperating governments. The first-order conditions for the equilibrium rate of investment, $\hat{I}^i$, are:

$$C^{i\prime}(\hat{I}^i) \geq U^i_{A^i}(\mathbf{A}); \;\; \hat{I}^i \geq 0; \;\; \hat{I}^i[C^{i\prime}(\hat{I}^i) - U^i_{A^i}] = 0 \tag{B.48}$$

Note that this differs from (B.18), since now each firm bases investments on its own value function. The equilibrium rates of taxes, $\hat{\tau}^i$, must satisfy the first-order conditions:

$$\tau^1 E(A^1)x^{1N}_{\tau^1} + \tau^2 E(A^2)x^{2N}_{\tau^1} = D^{1\prime} P^1_{\tau^1} + D^{2\prime} P^2_{\tau^1} - R^1_2 x^{2N}_{\tau^1} - R^2_1 x^{1N}_{\tau^1} \tag{B.49}$$

$$\tau^1 E(A^1)x^{1N}_{\tau^2} + \tau^2 E(A^2)x^{2N}_{\tau^2} = D^{1\prime} P^1_{\tau^2} + D^{2\prime} P^2_{\tau^2} - R^1_2 x^{2N}_{\tau^2} - R^2_1 x^{1N}_{\tau^2} \tag{B.50}$$

Insert (B.44) and solve for τ^1 and τ^2:

$$\hat{\tau}^i = D^{i\prime}\beta^{ii} + D^{j\prime}\beta^{ji} - \frac{1}{E(A^i)}R^i_j \tag{B.51}$$

Use (B.2), (B.1) and (B.44) to rewrite this condition to:

$$MB^i = MD^i, \tag{B.52}$$

as it is stated in the main text.

In case of emission standards the Hamilton-Jacobi-Bellman equations read:

$$rU^i(\mathbf{A}) = R^i - p^e\hat{\bar{e}}^i - C^i(\hat{I}^i) + U^i_{A^i}(\hat{I}^i - \delta^i A^i) + U^i_{A^j}(\hat{I}^j - \delta^i A^j) \tag{B.53}$$

$$rW(\mathbf{A}) = R^1 + R^2 - p^e\bar{e}^1 - p^e\hat{\bar{e}}^2 - C^1(\hat{I}^1) - C^2(\hat{I}^2) - D^1 - D^2 + W_{A^1}(\hat{I}^1 - \delta^i A^1) + W_{A^2}(\hat{I}^2 - \delta^i A^2) \tag{B.54}$$

for i=1,2. For $\hat{I}^i$, the equilibrium rate of investment, first-order conditions are given by (B.48). For $\hat{\bar{e}}^i$, the equilibrium standard, first-order conditions are:

$$(R^1_i + R^2_i)\frac{1}{E(A^i)} - p^e - D^{1\prime}P^1_{\bar{e}^i} - D^{2\prime}P^2_{\bar{e}^i} = 0 \tag{B.55}$$

for i=1,2. Remind (B.5) and use (B.2) and (B.1) to see that this can be rewritten to:

$$MD^i = MB^i, \tag{B.56}$$

as it is stated in the main text.

Open-loop investment

In case of an open-loop Stackelberg equilibrium, the first-order conditions on investment for competing firms are given by (B.29) to (B.31), but with different marginal benefits of abatement capital. That is, MBA^{iN} replaces MBA^{iC}. In case of taxes, $MBA^{iN} = -(p^e + \tau^i)x^{iN}E'(A^i) + R^i_j x^{jN}_{A^i}$. The last term stands for the marginal decrease in competing output and the effect this has on own firm revenues. In case of emission standards, $MBA^{iN} = -R^i_i\bar{e}^i\frac{E'(A^i)}{E(A^i)^2}$.

In case of taxes, the first-order conditions to the cooperating governments are in case of taxes given by (B.32) to (B.35) with x^{iN} instead of x^{iC} and MBA^{iN} instead of MBA^{iC}. Insert (B.44) in the adjusted versions of (B.29) to (B.35) and try the time-consistent solution $\nu^1(t) = \nu^2(t) = 0$. A contradiction is the result, namely:

$$\frac{R^j_i}{E(A^i)}x^{iN} + \frac{R^i_j(p^e + \tau^i)R^j_{ij}}{R^i_{ii}R^j_{jj} - R^i_{ij}R^j_{ij}} = 0 \tag{B.57}$$

Therefore, for scenario 2, the open-loop equilibrium with taxes is not time consistent. Insert (B.44) in (B.32) for i=1,2. Rearrange the system of two equations that results, to find:

$$\tau^i = D^{i\prime}\beta^{ii} + D^{j\prime}\beta^{ji} - \frac{1}{E(A^i)}R_i^j + \nu^i \frac{B^i}{E(A^i)} + \nu^j \frac{B^j}{E(A^i)} \tag{B.58}$$

where B^i and B^j denote the expressions:

$$B^i = \frac{MBA_{\tau^i}^{iN} x_{\tau^j}^{jN} - MBA_{\tau^j}^{iN} x_{\tau^i}^{jN}}{x_{\tau^i}^{iN} x_{\tau^j}^{jN} - x_{\tau^j}^{iN} x_{\tau^i}^{jN}} \tag{B.59}$$

and

$$B^j = \frac{MBA_{\tau^i}^{jN} x_{\tau^j}^{jN} - MBA_{\tau^j}^{jN} x_{\tau^i}^{jN}}{x_{\tau^i}^{iN} x_{\tau^j}^{jN} - x_{\tau^j}^{iN} x_{\tau^i}^{jN}} \tag{B.60}$$

Use (B.2), (B.1) and (B.44) to rewrite (B.58) to:

$$MB^i = MD^i + \nu^i \frac{1}{E(A^i)} B^i + \nu^j \frac{1}{E(A^i)} B^j \tag{B.61}$$

In case of emission standards, the open-loop first-order conditions to the cooperating governments are given by (B.37) to (B.40) with MBA^{iN} and x^{iN} instead of MBA^{iC} and x^{iC}. Try the solution $\nu^1(t) = \nu^2(t) = 0$ in the adjusted versions of (B.29) to (B.31) and (B.37) to (B.40). Some straightforward reordering gives:

$$R_i^j \bar{e}^i \frac{E'(A^i)}{E(A^i)^2} = 0 \tag{B.62}$$

This is not true so that there is no time-consistent solution to (B.29) to (B.31) and (B.37) to (B.40). Insert (B.2) and (B.1) in (B.37). The result is, that in equilibrium standards should be set such that:

$$MB^i = MD^i + \nu^1 \frac{\partial MBA^{1N}}{\partial \bar{e}^i} + \nu^2 \frac{\partial MBA^{2N}}{\partial \bar{e}^i}. \tag{B.63}$$

Scenario 3: Nash competition

In case of taxes, the first-order conditions for Nash-Cournot equilibrium choice of output by firms, $x^{iN}(\mathbf{A}, \tau)$ are given by (B.44). If these are inserted, the following differential game results:

manager of firm i:

$$\max_{I^i \geq 0} \int_0^\infty e^{-rt}[\Pi^i]dt$$

s.t. $\Pi^i = R^i(x^{1N}, x^{2N}) - (p^e + \tau^i)E(A^i)x^{iN} - C^i(I^i)$

$\dot{A}^i = I^i - \delta^i A^i$

government in country i:

$$\max_{\tau^i} \int_0^\infty e^{-rt}[G^i]dt$$

s.t. firm behaviour

In case of emissions standards, assume that both standards are binding in equilibrium, so that $x^i = \frac{\bar{e}^i}{E(A^i)}$ for i=1,2. The assumption (B.45) assures that this is a Nash equilibrium indeed. The differential game reduces to:

manager of firm i:

$$\max_{I^i \geq 0} \int_0^\infty e^{-rt}[\Pi^i]dt$$

s.t. $\Pi^i = R^i - p^e \bar{e}^i - C^i(I^i)$

$\dot{A}^i = I^i - \delta^i A^i$

government of country i:

$$\max_{\bar{e}^i} \int_0^\infty e^{-rt}[G^i]dt$$

s.t. firm behaviour

Feedback investment

The Hamilton-Jacobi-Bellman conditions for the game above are in case of taxes given by (B.46) and:

$$rW^i(\mathbf{A}) = R^i - p^e E(A^i)x^{iN} - C^i(\hat{I}^i) - D^i + W^i_{A^i}(\hat{I}^i - \delta^i A^i) + W^i_{A^j}(\hat{I}^j - \delta^i A^j) \tag{B.64}$$

with for $\hat{I}^i$, the equilibrium investment strategy, the first-order condition (B.48). The equilibrium rate of taxes, $\hat{\tau}^i$ must satisfy the first-order conditions:

$$(R^i_i - p^e E(A^i))x^{iN}_{\tau^i} = D^{i\prime} P^i_{\tau^i} - R^i_j x^{jN}_{\tau^i} \quad \text{for i=1,2} \tag{B.65}$$

Insert (B.44) and solve for τ^i to find:

$$\hat{\tau}^i = \frac{(D^{i\prime} P^i_{\tau^i} - R^i_j x^{jN}_{\tau^i})}{E(A^i)x^{iN}_{\tau^i}} \tag{B.66}$$

Then use (B.2) to rearrange this to:

$$\hat{\tau}^i = MD^i - D^{j\prime}\beta^{ji} + D^{i\prime}\beta^{ij}\frac{E(A^j)}{E(A^i)}\frac{x^{jN}_{\tau^i}}{x^{iN}_{\tau^i}} - R^i_j\frac{1}{E(A^i)}\frac{x^{jN}_{\tau^i}}{x^{iN}_{\tau^i}} \tag{B.67}$$

or with (B.1) also inserted:

$$MB^i = MD^i - R^i_j\frac{1}{E(A^i)}\frac{x^{jN}_{\tau^i}}{x^{iN}_{\tau^i}} + R^j_i\frac{1}{E(A^i)} - D^{j\prime}\beta^{ji} + D^{i\prime}\beta^{ij}\frac{E(A^j)}{E(A^i)}\frac{x^{jN}_{\tau^i}}{x^{iN}_{\tau^i}} \tag{B.68}$$

In case of standards on emissions, the conditions for a feedback equilibrium are (B.53) and:

$$rW^i(\mathbf{A}) = R^i - p^e\hat{\bar{e}}^i - C^i(\hat{I}^i) - D^i + W^i_{A^i}(\hat{I}^i - \delta^i A^i) + W^i_{A^j}(\hat{I}^j - \delta^i A^j) \tag{B.69}$$

First-order conditions on equilibrium investment policy, $\hat{I}^i$, are again given by (B.48). The first-order conditions for standards are:

$$R^i_i\frac{1}{E(A^i)} = p^e + D^{i\prime}\beta^{ii} \tag{B.70}$$

Use (B.2) and (B.1) to rewrite this to

$$MB^i = MD^i + R^i_j\frac{1}{E(A^i)} - D^{j\prime}\beta^{ji} \tag{B.71}$$

Open-loop investment

The first-order conditions for the firm manager are given by (B.29) to (B.31) with MBA^{iN} instead of MBA^{iC}. For the government in country i, in case of taxes, the first-order conditions read:

$$[R^i_i - p^eE(A^i)]x^{iN}_{\tau^i} + R^i_jx^{jN}_{\tau^i} - D^{i\prime}P^i_{\tau^i} - \nu^{i1}\frac{\partial MBA^{1N}}{\partial\tau^i} - \nu^{i2}\frac{\partial MBA^{2N}}{\partial\tau^i} = 0 \tag{B.72}$$

$$\dot{\mu}^{ii} = (r+\delta^i)\mu^{ii} - [R^i_i - p^eE(A^i)]x^{iN}_{A^i} - R^i_jx^{jN}_{A^i} + p^eE'(A^i)x^{iN} + D^{i\prime}P^i_{A^i}$$

$$+\nu^{i1}\frac{\partial MBA^{1N}}{\partial A^i} + \nu^{i2}\frac{\partial MBA^{2N}}{\partial A^i} \tag{B.73}$$

$$\dot{\mu}^{ij} = (r+\delta^i)\mu^{ij} - [R^i_i - p^eE(A^i)]x^{iN}_{A^j} - R^i_jx^{jN}_{A^j} + D^{i\prime}P^i_{A^j}$$

$$+\nu^{i1}\frac{\partial MBA^{1N}}{\partial A^j} + \nu^{i2}\frac{\partial MBA^{2N}}{\partial A^j} \tag{B.74}$$

$$\dot{\nu}^{ii} = -\delta^i\nu^{ii} + \frac{C^{i\prime}(I^i) - \mu^{ii}}{C^{i\prime\prime}(I^i)} \tag{B.75}$$

$$\dot{\nu}^{ij} = -\delta^i\nu^{ij} - \frac{\mu^{ij}}{C^{j\prime\prime}(I^j)} \tag{B.76}$$

$$\nu^{ii}(0) = 0 \tag{B.77}$$

$$\nu^{ij}(0) = 0 \tag{B.78}$$

for i=1,2. Insert the first-order condition on output, (B.44) in (B.72) and rearrange to find:

$$\tau^{iN} = D^{i\prime}\beta^{ii} + D^{j\prime}\beta^{ij}\frac{E(A^j)}{E(A^i)}\frac{x^{jN}_{\tau^i}}{x^{iN}_{\tau^i}} - R^i_j\frac{1}{E(A^i)}\frac{x^{jN}_{\tau^i}}{x^{iN}_{\tau^i}} + \frac{1}{E(A^i)x^{iN}_{\tau^i}}[\nu^{ii}\frac{\partial MBA^{iN}}{\partial\tau^i} + \nu^{ij}\frac{\partial MBA^{jN}}{\partial\tau^i}] \tag{B.79}$$

Here

$$\frac{\partial MBA^{iN}}{\partial\tau^i} = E'(A^i)[-x^{iN} - (p^e + \tau^i)x^{iN}_{\tau^i}] + R^i_{ij}x^{jN}_{A^i}x^{iN}_{\tau^i} + R^i_j x^{jN}_{A^i\tau^i} \tag{B.80}$$

and

$$\frac{\partial MBA^{jN}}{\partial\tau^i} = -(p^e + \tau^j)E'(A^j)x^{jN}_{\tau^i} + R^j_{ij}x^{iN}_{A^j}x^{jN}_{\tau_i} + R^i_j x^{iN}_{A^j\tau^i} < 0 \tag{B.81}$$

Use the definitions of marginal benefits and marginal damage to rewrite this to:

$$MB^i = MD^i - D^{j\prime}\beta^{ji} + R^j_i\frac{1}{E(A^i)} + D^{j\prime}\beta^{ij}\frac{E(A^j)}{E(A^i)}\frac{x^{jN}_{\tau^i}}{x^{iN}_{\tau^i}} - R^i_j\frac{1}{E(A^i)}\frac{x^{jN}_{\tau^i}}{x^{iN}_{\tau^i}} +$$

$$\frac{1}{E(A^i)x^{iN}_{\tau^i}}[\nu^{ii}\frac{\partial MBA^{iN}}{\partial\tau^i} + \nu^{ij}\frac{\partial MBA^{jN}}{\partial\tau^i}] \tag{B.82}$$

The solution $\nu^{11}(t) = \nu^{22}(t) = \nu^{12}(t) = \nu^{21}(t) = 0$ results in a contradiction if it is inserted into into the equilibrium conditions. It can be concluded that the open-loop Stackelberg equilibrium is not time consistent.

In case of emission standards, the first-order conditions to the government of country i read:

$$[R^i_i - p^e E(A^i)]\frac{1}{E(A^i)} - D^{i\prime}\beta^{ii} - \nu^{i1}\frac{\partial MBA^{1N}}{\partial\overline{e}^i} - \nu^{i2}\frac{\partial MBA^{2N}}{\partial\overline{e}^i} = 0 \tag{B.83}$$

$$\dot{\mu}^{ii} = (r + \delta^i)\mu^{ii} - R^i_i x^{iN}_{A^i} + \nu^{i1}\frac{\partial MBA^{1N}}{\partial A^i} + \nu^{i2}\frac{\partial MBA^{2N}}{\partial A^i} \tag{B.84}$$

$$\dot{\mu}^{ij} = (r + \delta^i)\mu^{ij} - R^i_j x^{jN}_{A^j} + \nu^{i1}\frac{\partial MBA^{1N}}{\partial A^j} + \nu^{i2}\frac{\partial MBA^{2N}}{\partial A^j} \tag{B.85}$$

$$\dot{\nu}^{ii} = -\delta^i\nu^{ii} + \frac{C^{i\prime}(I^i) - \mu^{ii}}{C^{i\prime\prime}(I^i)} \tag{B.86}$$

$$\dot{\nu}^{ij} = -\delta^i\nu^{ij} - \frac{\mu^{ij}}{C^{j\prime\prime}(I^j)} \tag{B.87}$$

$$\nu^{ii}(0) = 0 \tag{B.88}$$

$$\nu^{ij}(0) = 0 \tag{B.89}$$

for i=1,2. Insert the derivatives of $x^i = \frac{\overline{e}^i}{E(A^i)}$ to A^i and of MBA^{iN} to own and foreign standards and rearrange equation (B.83). Then, it follows that in equilibrium standards should be set such that:

$$R^i_i - p^e E(A^i) - R^j_i - D^{j\prime}\beta^{ji}E(A^i) - \nu^{ii}[R^i_{ii}\overline{e}^i\frac{E'(A^i)}{E(A^i)} + R^i_i E'(A^i)] + \nu^{ij}R^i_j\overline{e}^j\frac{E(A^i)}{E(A^j)}\frac{E'(A^j)}{E(A^j)} = 0 \tag{B.90}$$

Which can be rewritten to

$$MB^i = MD^i - D^{j\prime}\beta^{ji} + R^j_i\frac{1}{E(A^i)} + \nu^{ii}MBA^{iN}_{\overline{e}^i} + \nu^{ij}MBA^{jN}_{\overline{e}^i} \tag{B.91}$$

If the solution with $\nu^{ii}(t) = \nu^{ij}(t) = 0$ is inserted, a contradiction is obtained: From the conditions on each firm and government and specifically from (B.85) and (B.87), it follows that

$$R^i_j x^{jN}_{A^j} = 0 \tag{B.92}$$

is required in that case, which cannot be true for general functions. This shows that the open-loop Stackelberg equilibrium in case of emission standards is not time consistent.

Chapter 7

Environmental policy and investment in emission reduction, the case of international competition

In this chapter, we analyse firms' incentives to invest in emission reducing technology under emission taxes and emission standards. That is done in a context of imperfect competition, international rivalry and transboundary pollution. As a result there is strategic interaction between firms, between governments and between firms and governments.

A differential game model is used to analyse a duopoly, with each competitor situated in a different country. Governments balance trade interests with environmental concerns. If they impose the strict policy they prefer from an environmental point of view, this might harm home firms too much and decrease their profits substantially. It is assumed that governments do not want this, since they have a preference for high profits to be earned at home. This leads to downward distortions in environmental policy. At the same time, investments in emission reduction, once installed and paid for, lead to a cost advantage.

Firms base their decisions to invest on expected future policy. Due to the distortions in environmental policy, it is not a priori clear which type of instrument, taxes or standards, gives the best incentive to invest in emission reducing technologies, that is the incentive that is closest to the social optimum. In contrast to what is generally believed, it can happen that environmental taxes are less dynamically efficient than standards.

1 Introduction

International rivalry may seriously affect decisions on environmental policy, if governments value high profits for their home firms, besides high environmental quality. An extensive literature now exists on environmental policy making within a framework of international competition among firms[1] (among others Barrett (1994), Kennedy (1994), Markusen, Morey and Olewiler (1993), Rauscher (1994), Ulph (1996) and Ulph (1994), who also provides an overview). With oligopolistic international output markets, concern for the competitiveness of their home firms may lead governments to distort environmental policies. Such distortions are, in the absence of trade policy, a second best-policy for a government that wants both to reduce environmental damage and to improve home firms' profits.

The question arises what are the implications for firms' incentives to invest in emission reduction technologies. Governments have an interest in these investments, because they value low environmental damage in their countries. At the same time governments want to improve, or at least not reduce, the competitiveness of their home firms. Since firms decide on and pay for the investments to reduce their emissions, it is important to consider their incentives to invest. Without environmental policy, firms are assumed to neglect the environmental consequences of production. In that case, they have no reason to pay for investment to reduce emissions. As a result, governments have a different view about the "right" rate of investment in emission reduction technology than firms.

Environmental policy changes the incentives of firms, because it changes the (shadow-) price of emissions. The degree to which an environmental policy instrument gives firms the "right" incentives to invest in environmental improvements is also called its dynamic efficiency. Loosely speaking, the dynamic efficiency of a certain environmental policy concerns the incentives provided for polluters to invest in innovation and the direction in which innovative activities are steered. This chapter, however, analyses investment in emission reduction, rather than innovation. The chapter analyses the incentive to invest in emission reduction under different environmental policy instruments. That is done in a context of imperfect competition, international rivalry and transboundary pollution. As a result there is strategic interactions between firms, between governments and between a firm and its government.

Investment means that commitments for the future are made. These commitments play a role in strategic interactions. The interactions are formalized in a dynamic game model. This enables to be precise about the structure of decision making. It is shown that, beside immediate cost savings, strategic interactions are an important determinant of the rate of investment in emission reduction technologies.

Two environmental policy instruments are compared, namely emission taxes and emission standards. These two are chosen, since they represent policies directed at prices and policies directed at quantities. It turns out that the differences in commitment between them, have an impact on the incentives to invest in emission reductions.

Papers that analyse the relative dynamic efficiency of environmental policy instruments are among others Downing and White (1986), Jung, Krutilla and Boyd (1996), Magat (1978) and

[1] Part of this literature is reviewed in chapter 1.

Milliman and Prince (1989). Except for Magat, these papers analyse a static one country model with perfect competition. In that context, dynamic efficiency is easily defined to be the level of investment chosen by a social planner. Furthermore, strategic interactions do not play a role, due to the assumption of perfect competition. Moreover, there is no obvious reason to distort policy instruments from their pigouvian level.

The papers mentioned above concentrate on a comparison of the cost savings that can be obtained by firms, if they adopt a new technology that results in an overall decrease of marginal abatement costs to this firm. A good example is Downing and White (1986). They find that taxes give rise to overincentives to innovation, while emission standards lead to a (relatively stronger) underincentive. In case of emission standards, firms gain, because a new technology decreases abatement costs. In case of taxes, firms can adjust to a new level of emissions and their gain is larger, since it also includes some reductions in tax payments. If the original tax level is a reasonable approximation of social costs, then the incentive in case of taxes is close to the social optimum.

Requate (1994) includes output adjustments into the analysis. He compares taxes to auctioned marketable permits, under perfect competition. With output adjustments taken into account, taxes may provide more incentives to innovate to individual firms than marketable permits. It depends on marginal social damage, which instrument is to be preferred from a social welfare point of view. Malueg (1989) also points to the indeterminacy of dynamic incentives. He considers a case where the instrument is not set at the socially optimal level and compares emission standards to grandfathered marketable permits. Malueg shows that if permit prices do not coincide with marginal social damage, it cannot be concluded that permits provide more correct incentives for innovation than standards. A similar reasoning can be followed for taxes and standards. It can be concluded that if taxes are set below marginal social damage and the standard imposed on a firm implies a lower level of emissions than the tax, this standard gives better incentives than the tax. Malueg does not give an explanation why environmental policies are distorted. The analysis below provides such an explanation. It derives the distortions that can be expected under international rivalry and transboundary pollution.

When governments use open-loop strategies and firms apply open-loop investment strategies, results are in general inconclusive, as it was shown in the previous chapter. The inconclusiveness is due to the twofold effect of an increase in abatement capital. More abatement capital leads to less emissions per unit of output and therefore decreases total emissions given constant output. But, more abatement capital also leads to increased output and therefore more emissions. As a result, there is no general rule in what direction governments which prefer both to improve firm profits and to reduce environmental damage should steer abatement investments. Ulph and Ulph (1996) applies a multistage model. As noted results from such a model will be comparable to open-loop equilibria in the full dynamic model. It is therefore not a surprise that they find that both the question whether stricter environmental policy encourages or discourages firms' investment in emission reduction and the question in what direction governments distort their environmental policy has an answer that depends on the specification of the model. A crucial determinant is the shape of the function that models the relation between emissions and the amount of abatement capital.

When feedback investment and environmental policy strategies are applied, the direction of the distortions in policy instruments can be determined. If governments have no commitments to future levels of environmental policy, firms base their investment on their expectations about future policies. Hence governments cannot influence investment in abatement technology with current policies. Then the only distortions in policy levels are due to transboundary pollution and strategic trade effects.

The overall effect of all distortions on investments in emission reduction is difficult to predict. In particular it is shown below that it is not true that taxes are always more dynamically efficient than standards. That is, it is well possible that distorted standards result in investments that are closer to the social optimal rate than distorted taxes.

The model used in this chapter was introduced in chapter 6. Sections 6.1 to 6.4 in that chapter give the basis for the analysis below. The rest of this chapter is set up as follows. The next section gives equilibrium conditions for environmental policy for a social planner benchmark case and repeats the equilibrium conditions for the scenario of Nash competition between firms and governments given in chapter 6. The distortions in environmental policy are explained. Section 3 uses a counterexample to show that taxes do not always provide better dynamic incentives than standards. The result is clarified with the help of equilibrium conditions on investment in section 4. Section 5 concludes.

2 Equilibrium policy for different instruments

In this section, the first-order conditions for the equilibrium strategies of the government for the "social planner" benchmark are given and the conditions for the scenario with competition between firms and competition between governments are recapitulated from chapter 6.

Consider first the "social planner" solution as a benchmark case. Assume that in each country, the government completely controls its home firms. In that case it does not need environmental policy but decides directly on output and investment. In Nash equilibrium with the other country it then has to solve:

$$\max_{x^i, I^i} \int_0^\infty e^{-rt} \left[R^i(x^i, x^j) - p^e E(A^i)x^i - C(I^i) - D^i(P^i)\right] dt \tag{1}$$

$$\text{s.t. } \dot{A}^i = I^i - \delta A^i \tag{2}$$

$$\dot{A}^j = I^j - \delta A^j. \tag{3}$$

Equilibrium conditions for this dynamic game can be found in appendix A. A condition on equilibrium output, x^{iS}, is:

$$R_i^i(x^{iS}, x^{jS}) = (p^e + D^{i\prime}(P^i)\beta^{ii})E(A^i). \tag{4}$$

Given the choices of the other country, output is chosen such that marginal benefits equal marginal costs, which include environmental damage at home, but neglect damage done in the

other country.
Now consider the dynamic game set up in section 4 in chapter 6. Two governments set environmental policy and two firms compete in a Cournot duopoly. The government should set standards respectively taxes in Nash competition with the other government, such that the equilibrium behaviour of the two firms given these policies maximizes the government's objectives. A complete set of equilibrium conditions is given in appendix B in chapter 6.
An equilibrium condition for emission standards is:

$$R_i^i(x^{iN}, x^{jN}) = (p^e + D^{i\prime}(P^i)\beta^{ii})E(A^i) \tag{5}$$

This is similar to condition (4). But now the governments cannot directly control output and have to evaluate marginal benefits and costs at the rate of output, x^{iN}, that will be chosen by the firm if it satisfies the standard. Given a stock of abatement capital A and assuming that the standard, $\bar{e}$, is binding, it immediately follows that this output rate is equal to $\frac{\bar{e}}{E(A)}$.
In case of emission taxes, the comparable equilibrium condition is different:

$$R_i^i(x^{iN}, x^{jN}) = (p^e + D^{i\prime}(P^i)\beta^{ii})E(A^i) + [D^{i\prime}\beta^{ij}E(A^j) - R_j^i]\frac{x_{\tau^i}^{jN}}{x_{\tau^i}^{iN}} \tag{6}$$

Two distortionary effects are present, because firms have indirect influence on each others output through the market price. As a consequence, lower taxes in one country reduce equilibrium output in the other country and governments take that into account. For the government of country i it is advantageous to discourage foreign production by a downward distortion in its taxes. A downward distortion lowers foreign equilibrium output, which reduces damage from abroad (as represented by the term $D^{i\prime}\beta^{ij}E(A^j)\frac{x_{\tau^i}^{jN}}{x_{\tau^i}^{iN}}$) and shifts rents to its home firm (as represented by the term $-R_j^i\frac{x_{\tau^i}^{jN}}{x_{\tau^i}^{iN}}$). Both motives hence explain lower equilibrium taxes than would be set if equilibrium output in the foreign country was independent of home environmental policy like under environmental standards. Using firms' equilibrium condition on output, $R_i^i = (p^e + \tau^i)E(A^i)$, condition (6) can be rewritten to give:

$$\tau^i = D^{i\prime}\beta^{ii} + \frac{1}{E(A^i)}[E(A^j)D^{i\prime}\beta^{ij} - R_j^i]\frac{x_{\tau^i}^{jN}}{x_{\tau^i}^{iN}} \tag{7}$$

This shows that taxes are set below marginal damage, due to the two motives mentioned above.

3 Dynamic efficiency

Dynamic efficiency in this chapter refers to the efficiency of the rate of investment chosen by firms as evaluated by their government. Given the context of perfect foresight and intertemporal optimization, individual decision makers always choose solutions that are dynamically efficient from their point of view. Since it was assumed that firms and governments shared the same rate of time preference, differences therein are no reason for dynamic inefficiencies. With only one firm in each country, positive externalities related to investment, which are another reason for dynamic inefficiency, are also absent. However, as will be shown below, firms distort their investments for strategic reasons. They want to influence environmental policy and foreign output to their own advantage. Likewise, the government distorts its policy for strategic reasons, as it was shown in the previous section. As a result of these distortions, it may be expected that the investment choices of firms are not considered dynamically efficient by their government.

As a benchmark case, the path of investment that results from the social planner solution is defined to be dynamically efficient. This path would be chosen if the government could directly choose the investment rate. Note that the choice of this benchmark implies that dynamic efficiency is defined for an individual country, because the two social planners compete with each other to maximize the welfare of their own country. In Nash equilibrium, the stock of abatement capital in one country affects the output and investment decisions by the other country. Therefore, strategic interactions between the two social planners distort investment in abatement capital from the rate that is based on immediate reductions in environmental damage and production costs. Besides, since investment and output rates are set according to feedback strategies, investment in abatement capital is a commitment to higher future output rates (because $x^i_{A^i} > 0$) as well as a commitment to lower future investment rates (because $I^i_{A^i} < 0$).

When the decisions on output and investment are taken by independent firms subject to environmental policy, three types of strategic interaction are present. First, like the social planners in the benchmark case, firms compete with each other on the output market. Therefore, there is strategic interaction between firms. Second, there is strategic interaction between governments, because they set their environmental policies in competition with the other country. Third, when firms are aware that the equilibrium strategies of environmental policy are functions of the stock of abatement capital, they try to choose these stocks such that environmental policy is as lax as possible. That is, strategic interaction between firms and governments is another reason for distortions in the rate of investment.

The total result of all these interactions is not clear ex-ante. It certainly is not true that taxes are always more dynamically efficient than standards, which was derived in Downing and White (1986) in a static one country context with perfect competition. That is shown below with the help of a counterexample. It shows a case where the incentives to invest under regulation with standards are under certain circumstances closer to the benchmark case than the incentives under regulation with taxes. With explicit functional forms, the two-state-variable dynamic game can be approximated by a linear-quadratic dynamic game. The linear-quadratic game's steady state can be computed analytically, using a method explained in Fershtman and de Zeeuw (1992). For details on the approximation algorithm used here see chapter 3.

The example uses the following functional forms for i=1,2:

$$R^i = p^i * x^i \tag{8}$$

$$p^i = 10 - x^i - \alpha * x^j \text{ with } 0 < \alpha < 1 \tag{9}$$

$$D^i = 4 * P^i \tag{10}$$

$$C^i = 15I^i + {I^i}^2 \tag{11}$$

$$E(A^i) = 0.2 + (Eo - 0.2)e^{-b*A^i} \tag{12}$$

and the following values for the parameters:

$$p^e = 1,\ \beta^{ii} = \frac{1}{2},\ \beta^{ij} = \frac{1}{4},\ r = 0.08,\ \delta = 0.10,\ Eo = 2,\ b = 0.5$$

Thus, complete symmetry, linear damage, exponential abatement reduction and quadratic revenues are assumed. This example satisfies condition (8) in chapter 6, so that in the absence of environmental policy, $A^i = A^j = 0$ is the equilibrium steady state.
The parameter α describes the degree of substitutability between the products of the two firms. If $\alpha = 0$, the products are traded on separate markets and firms do not compete with each other. In that case, strategic interaction between firms is absent, whereas interaction between governments is confined to transboundary pollution. If $\alpha = 1$, the products are perfect substitutes and strategic effects on the output market are strongest.
For different values of α, table 1 shows the equilibrium steady-state stocks of abatement capital. This shows that indeed when strategic interaction through the output market is sufficiently strong, investment under taxes is further from the benchmark than investment under standards.

Table 1: Steady state stocks of abatement capital.

α	taxes	standards	social optimum
0	2.7	1.5	2.7
0.1	2.6	1.5	2.6
0.5	2.8	1.6	2.4
1.0	4.6	2.1	2.7

The effects of interaction between firms, distorted environmental policies and interaction between the firm and its regulator together result in either underinvestment or overinvestment when compared to the benchmark case. Section 4 describes these effects in more detail. If the products are close substitutes (α close to 1), strategic interaction between firms leads to large overinvestments in abatement technology in case of taxes. In contrast, under standards, investment in abatement is too low compared to the benchmark case. Since taxes lead to more distortion from the dynamically efficient rate of investment than standards, this gives the counterexample that was required.

Table 2: Steady state stocks of abatement capital, case of local pollution

α	taxes	standards	social optimum
0	2.7	1.5	2.7
0.1	2.6	1.5	2.6
0.5	2.7	1.6	2.4
1.0	4.3	2.1	2.6

When α is lower and strategic interaction in the output market is therefore less important, the order is reversed. For very low interaction or no interaction in the output market, taxes provide almost the right incentives for investment in abatement capital. In case of standards, there is underinvestment also for low α. This underinvestment is mainly due to the interaction between the firm an its regulator. Section 4 gives a more complete analysis of the various distortions that lead to under- or overinvestment.

For comparison, consider a case with only local pollution (see table 2). The interaction due to transboundary pollution disappears, so that remaining distortions are completely due to competition on the output market and strategic interaction between the firm and the government. Take the same functional forms as given in (8) to (12), with the same parameter values, except that $\beta^{ij} = 0$.

Benchmark investment is slightly lower in case of local pollution. That is, strategic interaction is a bit lower, but not much. Investment under taxes is also slightly lower. Note, however, that the numbers have only illustrative value, so that conclusions about the relative magnitude of effects can not be made from them. Under standards, investment is the same whether transboundary pollution is absent or present. That makes sense for this specific example with constant marginal damage, because pollution from abroad can only be affected through foreign output and under standards there is no way in which foreign output may be influenced by home investment.

When markets are separate ($\alpha = 0$), strategic interaction through the output market is absent. The two firms act as local monopolies. Only interaction through transboundary pollution and interaction between government and firms remains. A comparison of the transboundary case and the case with only local pollution shows that interaction between government and firms is more important than interaction through transboundary pollution for the parameter values that were used in this example. There is almost no difference between equilibrium investment with or without transboundary pollution when $\alpha = 0$. Due to interaction between firm and government, under standards there is underinvestment compared to the social benchmark. Taxes provide almost correct incentives, since under taxes, interactions between the firm and its government are small (see section 4).

4 The firm's incentive to invest

To analyse the results suggested by the example it is useful to consider the equilibrium conditions on investment in some detail. A decision maker that optimizes its investment decisions equals the user cost of capital, $(r+\delta)C'(I^i)$ to its expected marginal value. The latter is expressed by the marginal derivative to capital of the value function, $V^i_{A^i}$ and denotes how much an additional unit of capital contributes to future profits and possibly environmental damage. The marginal derivative is only well defined when the value functions are differentiable. That is assumed to be the case in the sequel. The value function V^i is recursively defined by the Hamilton-Jacobi-Bellman optimality condition on investment in country i:

$$rV^i(A^i, A^j) = \max_{I^i}[W^i - C^i(I^i) + V^i_{A^i}(I^i - \delta A^i) + V^i_{A^j}(I^j - \delta A^j)] \tag{13}$$

Here W^i is either equal to $R^i - (p^e + \tau^i)e^i$ respectively $R^i - p^e e^i$, when the firm decides on investment, or equal to $R^i - p^e e^i$, in the benchmark case, where the social planner directly sets investment. From (13) it follows that the first-order condition for equilibrium investment is:

$$-C'(I^i) + V^i_{A^i} \leq 0 \quad I^i \geq 0 \quad I^i[V^i_{A^i} - C'(I^i)] = 0 \tag{14}$$

Assume an interior solution and take the partial derivatives of (13) to find an expression for $V^i_{A^i}$. The general equilibrium condition on investment then becomes[2]

$$(r+\delta)C'(I^i) = (r+\delta)V^i{}_{A^i} = W^i_{A^i} + V^i_{A^j}I^j_{A^i} + V^i_{ii}(I^i - \delta A^i) + V^i_{ji}(I^j - \delta A^j) \tag{15}$$

where $V^i_{A^j}$, the partial derivative of V^i to A^j is given by:

$$V^i_{A^j} = \frac{1}{(r+\delta - I^j_{A^j})}[W^i_{A^j} + V^i_{ij}(I^i - \delta A^i) + V^i_{jj}(I^j - \delta A^j)] \tag{16}$$

Condition (15) is comparable to the condition $r = f_k$ that must hold for optimal investment in static models, without investment costs. In that case the user costs of capital equal the discount rate r, while here depreciation and adjustment costs must be taken into account.

The conditions (15) and (16) include terms with second-order partial derivatives of the value function to own and foreign abatement capital, V^i_{ii} and V^i_{ji}. These denote how net growth in own respectively the foreign capital stock is evaluated. In the steady state considered in the numerical example above, the last two terms are zero, since capital stocks do not grow in the steady state. Therefore, these terms are left out below. The marginal value of a unit of abatement capital is then its direct marginal contribution to welfare, $W^i_{A^i}$ plus its effect on foreign investments, $I^j_{A^i}$ as valued by $V^i_{A^j}$. The partial derivative of the value function to foreign capital, $V^i_{A^j}$, expresses the valuation of the foreign capital stock. Since investment decisions immediately lead to a different capital stock, $V^i_{A^j}$ is the right price at which to evaluate $I^j_{A^i}$. The value of $V^i_{A^j}$ in turn is given by the properly discounted effects of a change in foreign capital stocks on home welfare (see (16)).

[2]Here V^i_{ij} denotes the second-order partial derivative of V^i to A^i and A^j, etcetera.

For a social planner, $W^i = R^i - p^e e^i$ and the first-order condition on equilibrium investment is therefore[3]:

$$(r+\delta)C'(I^i) = -(p^e + D^{i'}\beta^{ii})E'(A^i)x^i + R^i_{x^j}x^j_{A^i} - D^{i'}\beta^{ij}E(A^j)x^j_{A^i} + V^i_{A^j}I^j_{A^i} \quad (17)$$

with

$$V^i_{A^j} = \frac{1}{(r+\delta - I^j_{A^j})}[R^i_{x^j}x^j_{A^j} - D^{i'}\beta^{ij}\frac{de^j}{dA^j}] \quad (18)$$

The marginal effect of abatement capital on home welfare consist of four parts. The immediate cost savings that result are the first part. Per unit of output, social costs (including environmental costs at home) are decreased by $-E'(A^i)(p^e + D^{i'}\beta^{ii})x^i$. Here x^i is the equilibrium rate of output. This is given by condition (4). The second and third part represent a rent-shifting effect and the change in damage from abroad, both due to changes in foreign equilibrium output. They are positive, so that strategic interaction in the output market leads to more investment than is justified if only direct cost savings are counted. Both effects disappear if there is no strategic interaction in the output market, that is, when $\alpha = 0$. The fourth term, $V^i_{A^j}I^j_{A^i}$, describes the valuation by the social planner of changes in foreign investment. If more abatement capital discourages foreign investment and the social planner in country i values this reduction positively, then this term also is an additional incentive to invest. That is the case in our example. $V^i_{A^j}$ as defined by (18) is determined by a rent-shifting effect (less foreign abatement capital decreases foreign output) and by the change in damage from abroad that is due to changes in foreign abatement capital.

The incentive to invest defined by equation (17) is the benchmark against which other incentives should be compared. Due to competition between governments, competition between firms and between firm and regulator, this benchmark cannot be obtained by regulation. There are too many distortions to regulate with only one environmental policy instrument.

Now consider the equilibrium conditions for investment for a firm under environmental policy. For an easier exposition, the terms that have to do with the interaction between the firm and its governments are suppressed at first. They are added later, so that the complete conditions are given by (23) and (25) on page (175) below. For standards as an instrument of policy, the first part of the condition on optimal investment for firm i, that competes with firm j and is regulated by the government i, is then given by:

$$(r+\delta)C'(I^i) = -(p^e + D^{i'}(P^i)\beta^{ii})E'(A^i)x^i + U^i_{A^j}I^j_{A^i} \quad (19)$$

where $U^i_{A^j}$ is given by:

$$U^i_{A^j} = \frac{1}{(r+\delta - I^j_{A^j})}[R^i_{x^j}x^j_{A^j}] \quad (20)$$

[3]leaving out the V_{ii} and V_{ij} terms.

Here equilibrium output is from condition (5). Standards provide strong output commitments, because when they are binding, it is always optimal for firms to produce exactly at rate $\frac{\bar{e}}{E(A)}$. Due to that commitment, strategic interaction through the output market is no longer possible. As a result, only immediate cost savings and effects on foreign investment contribute to the marginal benefits of an additional unit of abatement capital. Whether or not pollution is transboundary then makes no difference in case of standards, because terms with β^{ij} are absent from the equilibrium conditions. When $\alpha = 0$, interaction through the output market is absent and the firm attaches no value to changes in foreign output. Hence $U^i_{A^j}$ is zero in that case. Then only immediate social cost savings determine equilibrium investment in abatement under standards. Note that this is not true for the benchmark case, because the social planner, in contrast to the firm, values foreign output also for its effect on transboundary pollution. The term $D^{i'}\beta^{ij}E(A^j)x^j_{A^i}$ in (17) reflects this.

For taxes as an instrument of environmental policy, the incentive to invest contains strategic terms, since in that case, like in the benchmark case, the decision makers (now firms) can affect each other's output choices. Firms take environmental damage into account only as far as this is reflected by emission taxes. The first part of the equilibrium condition in case of taxes then is:

$$(r+\delta)C'(I^i) = -(p^e + \tau^i)E'(A^i)x^{iT} + R^i_j x^{jT}_{A^i} + U^i_{A^j} I^j_{A^i} \tag{21}$$

with x^{iT} the output rate chosen by firm i in case of taxes and $U^i_{A^j}$ given by:

$$U^i_{A^j} = \frac{1}{(r+\delta - I^j_{A^j})}[R^i_j x^j_{A^j}] \tag{22}$$

The first term again gives immediate cost savings that include the tax. This tax does not fully reflect environmental damage in country i, since the government distorts taxes to influence competition. From (6), taxes are set below marginal damage at home. Immediate cost savings are therefore lower than in the benchmark case and than in case of standards. The second term reflects strategic effects. Since in case of taxes, the foreign firm is more flexible, there is better possibility to influence its behaviour than in case of standards. Compared to the benchmark case, the term that reflects changes in damage from abroad is absent here, because the firm now decides on investment and does not value environmental damage. Instead the regulator distorts the tax. That is, however, a less effective way to reduce foreign output than direct distortions in investment.

In the above expressions for equilibrium investment under standards and taxes, a number of distortions are yet missing. These concern interactions between the firm and its regulator. Given feedback strategies of environmental policy, capital stocks at home influence both environmental policy at home and environmental policy abroad. The latter affects foreign output and therefore home damage and profits. Under standards, the interaction through foreign policy is the only way in which firms may influence foreign output choices. When these interactions are taken into account, conditions under standards are extended to:

$$(r+\delta)C'(I^i) = -(p^e + D^{i'}\beta^{ii})x^{iS}E'(A^i) + R^i_j x^j_{\bar{e}^j}\bar{e}^j_{A^i} + D^{i'}\beta^{ii}\bar{e}^i_{A^i} + U^i_{A^j} I^j_{A^i} \tag{23}$$

with

$$U^i_{A^j} = \frac{1}{(r+\delta - I^j_{A^j})}[R^i_j(x^j_{A^j} + x^j_{\bar{e}^j}\bar{e}^j_{A^j}) + D^{i'}\beta^{ii}\bar{e}^i_{A^j}] \tag{24}$$

The sign of $\bar{e}^i_{A^i}$ and $\bar{e}^i_{A^j}$ is derived in appendix C for the special case of a linear damage function. It turns out that $\bar{e}^i_{A^j} < 0$, which means that more foreign abatement capital induces a stricter environmental policy at home, that is a lower standard. An explanation could be that the government tries to induce the home firm to keep up with the foreign firm and invest in abatement technology. Note that transboundary pollution cannot explain the sign of $\bar{e}^i_{A^j}$ in this case, since linear damage was assumed. Linear damage implies that changes in A^j do not change the marginal damage at home, measured by $D^{i'}$, of emissions from the home firm. The marginal effect on foreign standards of changes in domestic abatement capital, that is, $\bar{e}^j_{A^i}$, is hence an incentive to invest more, because more investment is a way to reduce foreign standards and discourage foreign output. The sign of $\bar{e}^i_{A^i}$ is indeterminate. It depends on which of two effects dominates. One effect is the incentive for the government to set a stricter standard, since it is cheaper for a firm with more abatement capital to obey this standard. The other effect is the incentive to set a laxer standard, since with a more environmental friendly technology, output causes less damage. When the total effect is negative, there is an incentive for the firm not to invest too much in environmental friendly technology, because this would induce stricter policy.

For taxes as an instrument of environmental policy the equilibrium conditions become:

$$(r+\delta)C'(I^i) = -(p^e + \tau^i)E'(A^i)x^{iT} + R^i_j(x^{jT}_{A^i} + x^{jT}_{\tau^i}\tau^i_{A^i} + x^{jT}_{\tau^j}\tau^j_{A^i}) -$$

$$E(A^i)x^{iT}\tau^i_{A^i} + U^i_{A^j}I^j_{A^i} \tag{25}$$

with

$$U^i_{A^j} = \frac{1}{(r+\delta - I^j_{A^j})}[R^i_j(x^{jT}_{A^j} + x^{jT}_{\tau^i}\tau^i_{A^j} + x^{jT}_{\tau^j}\tau^j_{A^j}) - E(A^i)x^{iT}\tau^i_{A^j}] \tag{26}$$

Effects on foreign output now include indirect effects through policy adjustments. The additional terms in (25) and (26) reflect effects on home environmental policy. The signs of these effects are derived in appendix C, again for the special case where the damage function is linear. Then $\tau^i_{A^i} < 0$ provided that a reasonable condition is satisfied, while the sign of $\tau^i_{A^j}$ is undetermined (For details see the appendix). A rough explanation why more domestic abatement capital leads to lower domestic taxes, is that emission per unit of output is lower if the abatement capital stock is higher. With lower emissions per unit, the marginal damage per unit of output decreases, while its marginal benefits stay the same. The rate of output preferred by the government, which balances firm profits and environmental quality, is therefore higher. As a result, taxes are set at a lower level. Of course, lower taxes are appreciated by the firm and the effect on taxes is an additional incentive to invest. If $\tau^i_{A^j} > 0$, more foreign abatement capital leads to higher taxes, which the firm does not like. It hence tries to reduce foreign investment. An explanation why foreign abatement capital increases taxes could be that the government tries to stimulate its home firm to invest in abatement as well, in order to keep up with the foreign firm. However, parameter values such that $\tau^i_{A^j} < 0$ also exist. Then other effects dominate the effect that the government wants to stimulate its home firm to keep up with the foreign firm.

Besides these direct effects of capital stocks on equilibrium policies, the expressions (23) and (25) include more indirect effects of investment in abatement capital on environmental policy. In total, these expressions contain a lot of direct and indirect effects of investment. It is not clear ex-ante what are the signs of all distorting terms, neither whether the final effect will be too much or too less incentives to invest compared to the benchmark case. In the numerical example above, it depends on the degree of competition between firms, whether taxes provide almost correct incentives or lead to too much investment compared to the benchmark case. Standards, in contrast, in the example lead to underinvestment. When output competition is large, standards as a result are closer to the benchmark case than taxes.

In static analyses, like in Downing and White (1986) standards give an underincentive, while taxes give a more or less right incentive to invest. This however was derived in a case with local pollution, perfect competition and no adjustments. In the context of the expressions above that implies that it is assumed that $\beta^{ij} = 0$, $x^i_{A^i} = x^i_{A^j} = 0$ and $R^i_j = 0$. Then (17) becomes:

$$(r+\delta)C(I^i) = -(p^e + D^{i'}\beta^{ii})E'(A^i)x^{iS} + V^i_{A^j} I^j_{A^i} \tag{27}$$

with $V^i_{A^j}$ equal to zero, since no reasons for foreign abatement capital to have any effect on home welfare are included in the model assumptions. Thus, the incentive to invest in abatement capital only consists of direct costs savings. Under standards, since output is assumed to be constant, the only cost savings are reduced payments for e^i:

$$(r+\delta)C'(I^i) = -p^e x^{iSt} E'(A^i). \tag{28}$$

The term $U^i_{A^j} I^j_{A^i}$ is left out since, like in the benchmark case, $U^i_{A^j} = 0$. Under taxes, the equilibrium condition for investment is given by

$$(r+\delta)C'(I^i) = -(p^e + \tau^i)E'(A^i)x^{iT} \tag{29}$$

and again $U^i_{A^j} = 0$.

It can immediately be seen from these three conditions, that indeed standards provide too little incentive, while taxes provide the correct incentive, provided that they are set according to equation (6), which here simplifies to $\tau^i = D^{i'}\beta^{ii}$. It is clear that these conditions assume quite rigid behaviour, with almost no adjustment possibilities. The conditions above, (25) and (23), are the other extreme, where due to a lot of adjustment possibilities, all conceivable variations are possible. From the counterexample it follows that the neglect of these variations may result in misleading conclusions regarding the dynamic efficiency of policy instruments. In the presence of strategic interaction, there is not one instrument, either taxes or standards, that always is more dynamically efficient than the other.

5 Conclusions

A dynamic game model of an international duopoly was used to describe the environmental policy setting by national governments that balance environmental targets and home firm competitiveness. When pollution is transboundary and trade is characterized by strategic interaction there are several reasons for regulators to deviate from the pigouvian level of environmental policy. Quantity based instruments, in the form of emission standards, were compared to price based instruments in the form of emission taxes. Under emission standards firm's commitment to a certain output level is greater than under taxes. Therefore, there is less possibility to affect the output choices of the competitor with standards than with taxes. This leads to different equilibrium output and emission levels under both instruments.

Attention focused on the dynamic efficiency of both instruments. Within the specific setting that we used, with much more adjustments than in the usual case, no instrument is dominating the other in this respect. Hence, if decision makers act rationally and take all direct and indirect effect of their decisions into account, it is not true that standards are always less dynamically efficient than taxes. It depends on the initial level of technology, on the shape of revenue functions and on the effects of a more environmental friendly technology on operating costs which instrument is more dynamically efficient.

In the model, abatement investments decrease operating costs, since they tend to increase the efficient use of inputs. As a result, if the price of the polluting input is high enough, firms already invest in abatement technology. However, when the price of the polluting input is not high enough, the net present value of abatement investments is negative, so that firms do not invest in abatement, unless they are subject to some environmental policy. This is the case that is considered in the main part of the analysis

The analysis assumes perfect foresight so that firms are indeed able to base their investment decisions on expectations about the future. In reality such foresight is absent and it may be the case that current taxes are used by firms to predict future taxes. In that case the open-loop equilibria discussed in chapter 6 may give a better description than feedback equilibria.

Alternatively, it may be compared what happens when firms are myopic and base their decisions on current tax levels, as is done in Magat (1978). In Magat's paper firms choose between investment that improves their productivity and investment that improves the environmental friendliness of their production process. Taxes and standards have a different effect on the degree to which firms invest in abatement. They lead to a bias in investment either towards abatement or towards productive investment. The direction of the bias under each instrument changes with the flexibility of the production function, that is with the possibility for the firm to switch factors from productive to abatement activities. These results are in agreement with our results in the sense that there is not one instrument that can be said to be the instrument that always leads to most investment in abatement technology.

The interaction between the firm and its regulator is an important part of the strategic incentives to invest in abatement capital. Since no information asymmetry was assumed, this interaction differs from principal-agent problems. The regulator does not need to extract information from the firm, but it reacts to the commitments made by the firm. The firm can foresee what

environmental policy it will face in the future. Cadot and Sinclair-Desgagné (1995) analyse a related problem. In their paper, a firm is faced with a regulator that will impose a prescribed standard at some, yet uncertain, future time. Since the regulators pay-off also depends on the profits of the firm, it reacts to the firm's efforts to develop a new, less polluting, technology. This leads to a game, where the threat of regulation gives the firm incentives to indeed invest in abatement technology. Firms cannot exactly foresee future environmental policy, since they may or may not be regulated. If governments weigh firm profitability very high, the equilibrium may be a situation with no regulation and no investment. In the model analysed in this chapter, an analogous situation exists, when for zero stocks of abatement capital, still the weight attached to firm profits is so high that rather than environmental taxes, regulators introduce subsidies on emissions, because strategic motives dominate environmental damage. In case of standards, that would imply an equilibrium with no binding standards.
An important extension would be to include stock damage of pollution, so that a realistic environmental damage function, instead of an evaluation of policy targets, could be used for the trade off between firm profits and environmental benefits. However, this would introduce at least a third state variable to the problem, which makes analysis very complicated.

A Equilibrium conditions for the social planner case

Equilibrium output

The first-order conditions for optimal choice of output for a social planner in country i are:

$$R_i^i \leq (p^e + D^{i'}\beta^{ii})E(A^i) \tag{A.1}$$

$$x^i \geq 0 \tag{A.2}$$

$$x^i[R_i^i - (p^e + D^{i'}\beta^{ii})E(A^i)] = 0 \tag{A.3}$$

All three conditions must hold for i=1,2. Nash equilibrium results in output choices $x^{1S}(A^1, A^2)$ and $x^{2S}(A^1, A^2)$. Assume that an interior solution with both outputs positive is reached.
If the optimal output choices are inserted, the following differential game results.

$$\max_{I^i} \int_0^\infty e^{-rt}\,[R^i(x^{1S}, x^{2S}) - p^e E(A^i)x^{iS} - C^i(I^i) - D^i(x^{1S}, x^{2S})]dt \tag{A.4}$$

$$\dot{A}^i = I^i - \delta A^i \quad \text{i=1,2} \tag{A.5}$$

$$I^i \geq 0 \quad \text{i=1,2} \tag{A.6}$$

Equilibrium investment

The Hamilton-Jacobi-Bellman conditions for the game (A.4) to (A.6) are:

$$rV^i(A^1,A^2) = R^i - p^e E(A^i)x^{iS} - C^i(\hat{I}^i) - D^i + V^i_{A^i}(\hat{I}^i - \delta A^i) + V^i_{A^j}(\hat{I}^j - \delta A^j) \tag{A.7}$$

Here V^i is the value function of the social planner in country i. $\hat{I}^i$ is the equilibrium rate of investment in emission reduction derived from the condition that $\hat{I}^i$ should maximize the right-hand side of (A.7):

$$C^{i\prime}(\hat{I}^i) \geq V^i_{A^i}(A^1,A^2);\ \ \hat{I}^i \geq 0;\ \ \hat{I}^i[C^{i\prime}(\hat{I}^i) - V^i_{A^i}] = 0 \tag{A.8}$$

The equilibrium rate of output is given by equation (A.1) above, that must hold with equality in an interior solution. Assume that the value function is differentiable and take the partial derivatives to A^i, respectively A^j. This gives the following expressions, where the first-order conditions on investment and output, (A.8) and (A.1) are inserted.

$$(r+\delta)V^i_{A^i} = -(pe+D^{i\prime}\beta^{ii})E'(A^i)x^{iS} + R^i_j x^j_{A^i} + V^i_{A^j} I^j_{A^i} - D^{i\prime}\beta^{ij}E(A^j)x^j_{A^i} + V^i_{ii}(\hat{I}^i-\delta A^i) + V^i_{ij}(\hat{I}^j-\delta A^j) \tag{A.9}$$

$$(r+\delta)V^i_{A^j} = -D^{i\prime}\beta^{ij}E'(A^j)x^{jS} + R^i_j x^j_{A^j} - D^{i\prime}\beta^{ij}E(A^j)x^j_{A^j} + V^i_{ij}(\hat{I}^i-\delta A^i) + V^i_{jj}(\hat{I}^j-\delta A^j) \tag{A.10}$$

B Equilibrium investment with environmental policy

First some notation is introduced. Let $\mathbf{A}$, τ, $\mathbf{x}$ and $\overline{\mathbf{e}}$ denote the vectors (A^i,A^j), (τ^i,τ^j), (x^i,x^j) and $(\overline{e}^i,\overline{e}^j)$ respectively. Let U^i_i denote the partial derivative of U^i to A^i and define W^i_i analogously. The Hamilton-Jacobi-Bellman conditions for the game above are in case of taxes:

$$rU^i(\mathbf{A}) = R^i - (p^e + \hat{\tau}^i)E(A^i)x^{iN}(\hat{\tau},\mathbf{A}) - C^i(\hat{I}^i) + U^i_i(\mathbf{A})(\hat{I}^i - \delta A^i) + U^i_j(\mathbf{A})(\hat{I}^j - \delta A^j) \tag{B.1}$$

$$rW^i(\mathbf{A}) = R^i - p^e E(A^i)x^{iN}(\hat{\tau},\mathbf{A}) - C^i(\hat{I}^i) - D^i(P^i(\mathbf{x},\mathbf{A})) + W^i_i(\mathbf{A})(\hat{I}^i-\delta A^i) + W^i_j(\mathbf{A})(\hat{I}^j-\delta A^j) \tag{B.2}$$

In case of standards on emissions, the conditions for a feedback equilibrium are:

$$rU^i(\mathbf{A}) = R^i - p^e\overline{e}^i - C^i(\hat{I}^i) + U^i_i(\mathbf{A})(\hat{I}^i - \delta A^i) + U^i_j(\mathbf{A})(\hat{I}^j - \delta A^j) \tag{B.3}$$

$$rW^i(\mathbf{A}) = R^i - p^e\overline{e}^i - C^i(\hat{I}^i) - D^i(P^i(\overline{\mathbf{e}})) + W^i_i(\mathbf{A})(\hat{I}^i - \delta A^i) + W^i_j(\mathbf{A})(\hat{I}^j - \delta A^j) \tag{B.4}$$

Assume that the value functions are differentiable, and partially differentiate $U^i(A^i,A^j)$ to A^i and A^j respectively. This gives the following expressions for the marginal value of a unit of own and foreign abatement capital. Here the optimality conditions on output, investment and taxes, which can be found in section 6.B, have been inserted.

$$(r+\delta)U_i^i = -(pe+\hat{\tau^i})E'(A^i)x^{iN} + R_j^i(\frac{\partial x^{jN}}{\partial A^i} + \frac{\partial x^{jN}}{\partial \tau^i}\frac{\partial \tau^i}{\partial A^i} + \frac{\partial x^{jN}}{\partial \tau^j}\frac{\partial \tau^j}{\partial A^i}) -$$

$$E(A^i)x^{iN}\frac{\partial \tau^i}{\partial A^i} + U_j^i \hat{I^j}_{A^i} + U_{ii}^i(\hat{I^i} - \delta A^i) + U_{ij}^i(\hat{I^j} - \delta A^j) \tag{B.5}$$

and

$$U_j^i = \frac{1}{r+\delta-\hat{I^j}_{A^j}}[R_j^i(\frac{\partial x^{jN}}{\partial A^j} + \frac{\partial x^{jN}}{\partial \tau^i}\frac{\partial \tau^i}{\partial A^j} + \frac{\partial x^{jN}}{\partial \tau^j}\frac{\partial \tau^j}{\partial A^j}) - E(A^i)x^{iN}\frac{\partial \tau^i}{\partial A^j} +$$

$$U_{ij}^i(\hat{I^i} - \delta A^i) + U_{jj}^i(\hat{I^j} - \delta A^j)] \tag{B.6}$$

For standards, these expressions read:

$$(r+\delta)U_i^i = -(pe + D^{i\prime}\beta^{ii})E'(A^i)x^{iN} + R_j^i\frac{\partial x^{jN}}{\partial \bar{e}^j}\frac{\partial \bar{e}^j}{\partial A^i} + D^{i\prime}\beta^{ii}\bar{e}^i_{A^i} + U_j^i\hat{I^j}_{A^i} +$$

$$U_{ii}^i(\hat{I^i} - \delta A^i) + U_{ij}^i(\hat{I^j} - \delta A^j) \tag{B.7}$$

and

$$U_j^i = \frac{1}{r+\delta-\hat{I^j}_{A^j}}[R_j^i(\frac{\partial x^{jN}}{\partial A^j} + \frac{\partial x^{jN}}{\partial \bar{e}^j}\frac{\partial \bar{e}^j}{\partial A^j}) + D^{i\prime}\beta^{ii}\frac{\partial \bar{e}^i}{\partial A^j} + U_{ij}^i(\hat{I^i} - \delta A^i) + U_{jj}^i(\hat{I^j} - \delta A^j)] \tag{B.8}$$

Also here, optimality conditions on output, investment and standards have been inserted. These can again be found in section 6.B.

C The effect of abatement investment on environmental policy

To find the signs of $\bar{e}^i_{A^i}$, $\bar{e}^i_{A^j}$, $\tau^i_{A^i}$ and $\tau^i_{A^j}$, consider the first-order conditions that were derived in chapter 6 for the optimal policies under feedback strategies:

$$(R_i^i - p^e E(A^i))x_{\tau^i}^{iN} = D^{i\prime}\beta^{ii}E(A^i)x_{\tau^i}^{iN} + D^{i\prime}\beta^{ij}E(A^j)x_{\tau^i}^{jN} - R_j^i x_{\tau^i}^{jN}, \tag{C.1}$$

for taxes and

$$R_i^i\frac{1}{E(A^i)} = p^e + D^{i\prime}\beta^{ii}, \tag{C.2}$$

for standards. Take the derivatives of these equations for i=1,2 and assume that $D^{i''} = 0$ and that third derivatives of the revenue function can be neglected. Assume as well that $R^i_{jj} = R^j_{ii} = 0$. Then two systems of equations can be derived, that can be solved to give:

$$\bar{e}^i_{A^i} = E'(A^i)[\frac{\bar{e}^i}{E(A^i)} + \frac{E(A^i)(p^e + D^{i'}\beta^{ii})R^j_{jj}}{R^j_{jj}R^i_{ii} - R^i_{ij}R^j_{ij}}] \tag{C.3}$$

$$\bar{e}^i_{A^j} = -E'(A^j)\frac{E(A^i)R^i_{ij}(p^e + D^{j'}\beta^{jj}}{R^i_{ii}R^j_{jj} - R^i_{ij}R^j_{ij}} < 0 \tag{C.4}$$

and

$$\tau^i_{A^i} = \frac{E'(A^i)R^i_{ii}R^j_{ij}F}{E(A^i)(R^j_{jj}R^i_{ii}F^2 - {R^i_{ij}}^3{R^j_{ij}}^3)}[(p^e + D^{i'}\beta^{ii})R^i_{ij}R^j_{jj} - \frac{R^i_j}{E(A^i)}F + D^{i'}\beta^{ij}\frac{E(A^j)}{E(A^i)}F], \tag{C.5}$$

with $F = R^i_{ii}R^j_{jj} - 2R^i_{ij}R^j_{ij}$. This is negative provided that $F > 0$ and $(R^i_{ii}R^j_{jj}F^2 - {R^i_{ij}}^3{R^j_{ij}}^3) > 0$, which requires own effects (R^i_{ii} and R^j_{jj}) to dominate foreign effects (R^i_{ij} and R^j_{ij}) to a strong enough degree.

$$\tau^i_{A^j} = -\frac{E'(A^j){R^j_{ij}}^2{R^i_{ij}}^2}{E^i(R^i_{ii}R^j_{jj}F^2 - {R^i_{ij}}^3{R^j_{ij}}^3)}[-(p^e + D^{j'}\beta^{jj}){R^i_{ij}}^2 - D^{i'}\beta^{ij}(R^i_{ii}R^j_{jj} -$$

$$R^i_{ij}R^j_{ij}) - \frac{R^j_i}{E(A^j)}\frac{{R^i_{ij}}^3}{R^i_{ii}} + D^{j'}\beta^{ji}\frac{E(A^i)}{E(A^j)}\frac{{R^i_{ij}}^3}{R^i_{ii}}]. \tag{C.6}$$

Given that $F > 0$ and $(R^i_{ii}R^j_{jj}F^2 - {R^i_{ij}}^3{R^j_{ij}}^3) > 0$, $\tau^i_{A^j}$ is positive provided that the two second terms in the part between square brackets dominate the two first terms. If that is not the case and the two first terms dominate, then $\tau^i_{A^j} < 0$.

Chapter 8

Conclusions

1 Motivation

Environmental problems often have international aspects and this thesis analyses some of these aspects through the use of dynamic models. A recurrent theme is the comparison of the environmental policy instruments taxes and standards. There is a tradition of literature that compares these two instruments, when applied to solve environmental problems (see Bohm and Russell, 1985). Most of the advantages and disadvantages have become textbook material (see, for instance, Pearce and Turner, 1990). Usually, taxes and standards are compared on four aspects: their allocative efficiency, their effectiveness, their dynamic efficiency and their transaction costs. A fifth obvious difference between taxes and standards is that under taxes polluters must pay these taxes. Regulators at the same time receive the tax revenues, which they may redistribute. Taxes and standards therefore have different distributional implications. These comparisons take place in a closed economy setting. An open economy with international trade introduces new aspects. Pollution may cross borders, while governments are not able to directly regulate the behaviour of foreign polluters. Moreover, polluters may have international competitors. The regulation of polluters then affects their position in the international market. The comparison of taxes and standards in case of international trade is especially interesting when trade can be considered as oligopolistic competition on an international market. The market participants are aware of each other's behaviour and will deliberately try to influence the actions of others, that is, there is strategic interaction.

It was explained in the introduction (chapter 1) how oligopolistic competition between polluters, together with the interest of governments in the profits earned by domestic firms, explains the trade-strategic distortion of environmental policy instruments. Decisions on environmental policy influence the position of regulated firms in the international market. If governments care about the profits of domestic firms, that in turn affects their decisions about environmental policy; they distort policy for trade-strategic reasons. An interesting question is whether the type of environmental policy instrument makes any difference.

Ulph (1992) compares taxes and standards in a multistage game, which models firms which strategically trade on an international market. The model is a game consisting of three stages: first governments decide to apply taxes or standards, then firms choose their capital stocks and finally firms decide on output. The chosen instrument of environmental policy changes the character of strategic interaction among the firms.[1] Ulph finds that the government prefers standards because domestic firm earns more profits under standards than under taxes while the same environmental target is realized.

The research in this thesis was motivated by that result and by the observation that the problem at hand has many dynamic aspects: Investment in capital and the accumulation of pollution in the environment are inherently dynamic processes that take place over time. An investment at some point in time has implications for future production possibilities. Emissions at some moment imply an increase in pollution, which mostly assimilates only gradually.

Furthermore, most of the interactions involved, those between firms on the international market for output, those between governments and those between firms and governments do not involve once-and-for all decisions, but take place over a longer time period. That is, the interactions are a repeated process of action and reaction. Decision strategies may differ in their adjustment possibilities and therefore determine how players can react to the actions of other players. The more flexibility, the more interaction and the more important it is to use dynamic models, that allow for such continuous interaction. The research in this thesis therefore applies differential game models of the interactions between firms and governments. Such models give insight into the various interactions that occur and the effect of the different decision strategies which players adopt.

The price that must be paid for the precise modelling of the dynamics and for analytical tractability, is that the model has to be relatively simple in other aspects. In particular, this implies that chapters 2, 3, 6 and 7 model duopolies with each duopolist located in a different country, while consumers are assumed to be in a third country. The duopolists are regulated, each by its own government, which values only environmental quality and home-firm profits.[2] Furthermore, perfect foresight, complete information, interior solutions and no transaction costs are assumed. Tax revenues are redistributed to polluters by lump-sum transfers.

These simplifications imply that, with one firm in each country, the differences between taxes and standards in allocative efficiency cannot be studied. Furthermore, the model is not suitable to analyse differences between policy instruments in transaction costs, in effectiveness or in distributive effects. These aspects are abstracted away, to concentrate on the main theme of this thesis: issues of dynamic efficiency and distortions for trade strategic reasons. Dynamic efficiency is addressed especially in chapter 7. The comparison of standards and taxes is the subject of chapters 2, 3, 6 and 7, where the latter two chapters include transboundary pollution. A third environmental policy instrument, namely marketable permits, is considered in chapter 4. Chapter 5 focuses on the dynamics of transboundary stock pollution in a model

[1] Other papers use similar multistage models to analyse environmental policy target choice for given policy instruments in a setting of international rivalry. This literature and the strategic trade literature which forms the background for it has been reviewed in chapter 1.

[2] These same assumptions are common in multistage analyses of strategic trade models as well.

that abstracts from firm behaviour under government policy, but instead models the choices made by countries which have to decide on their emissions.

A common result is the following: There are differences between emission taxes and emission standards, if these instruments are applied by governments that engage in environmental policy competition. However, it is not true that one of these instruments is unambiguously 'better' than the other, in the sense that it results in higher values of the social objective function. Which instrument is 'better' depends among other factors on the type of investment strategy which the regulated firms apply.

2 A dynamic model of environmental policy competition

This section summarizes the intuition behind the general result stated above. With the help of the concept of commitment, it is explained why different equilibria result from different assumptions about either the type of investment strategy or the type of environmental policy instrument.

To explain environmental policy competition, commitment is a useful concept. The rationale for the government to interfere actively with the international competitiveness of its home firms is that, in case of trade by oligopolists, the government has the ability to give its home firms commitment and therefore a strategic advantage. Commitment means that the firm gains the ability to bind its hands, which is to its advantage in the competition with other firms on the output market. For example, if competition takes the form of Cournot competition, then commitment to a high level of output means that the reaction curve of the firm is shifted out. Governments could give their firm such commitments directly by export subsidies, but also indirectly by distorted policies, for instance environmental policies. If the degree of commitment is well chosen, the firm can earn more profits. To determine the appropriate degree of commitment, the government must balance the cost to provide the commitment with the additional profits that firms can earn with it.

For a given environmental target, the government prefers the policy instrument which guarantees the highest profits for its firm. If the target of environmental policy is not given, the government chooses the tax or the standard so that an objective function, that consists of both environmental damage and home firm profits, is maximized. Similar incentives exist for the other country. An equilibrium can be determined where all players act strategically.

How investment strategies influence the outcomes is best understood through a dynamic analysis of the implications of different investment strategies. The degree of commitment to the use of production factors or to output is the relevant factor. Some strategies imply quick reactions to changing circumstances, while others imply a greater rigidity. Under open-loop strategies of investment the rate of investment is fixed from the beginning and the firm will not change this in reaction to the actions of others. Under feedback strategies of investment the rate of investment depends on the current situation, and the firm adjusts its investment in reaction to changes in both its own and the foreign stock of capital.[3]

The environmental policy instruments of emission taxes and standards differ in their commitment effects. An emission standard implies a strong commitment to the use of the polluting input. For marginal changes in other's actions, the firm will not change the amount of the polluting input. In contrast, an emission tax does not give any commitment to the quantities of the polluting input.

Under open-loop strategies of investment then, standards are a preferred choice of instrument, because they give the firm a strong commitment and make it less sensitive to strategic interaction. As a result, if both governments apply standards, in equilibrium investment for strategic reasons is lower than if both governments apply taxes. This result, which is derived in chapter 2 confirms what was found in a multistage model.

In chapter 3 it is shown that this result depends on the assumption of open-loop investment strategies. Under feedback investment strategies such clearcut results are no longer possible. It is not necessarily true that taxes are 'better' than standards or vice versa. A numerical example in chapter 3 shows how this depends on the parameters of the model.

Chapters 5, 6 and 7 extend the analysis with transboundary pollution, which is an independent reason for distortions in environmental policy. Without international cooperation, transboundary pollution is, by itself, a cause of suboptimal levels of environmental policy.

Chapter 5 addresses the role of production forests in the transboundary pollution problem of global warming. Trees assimilate carbondioxide while they grow. Harvested trees release their carbon content at some point in time. If a country owns a large production forest, it may want to include the carbon dynamics of its forest when net emissions of carbondioxide are determined in international agreements. This chapter compares the global social optimum and the noncooperative outcome for the case where the role of forests is included with the outcomes for the case where this role is ignored. It turns out that for a large enough initial forest, harvest should always be lower if net emissions count and if carbon assimilation and emission of forests is included.[4]

Transboundary pollution furthermore increases the distortions that are due to strategic trade, since the additional damage from laxer policies is valued less if this damage is partly abroad. With transboundary pollution, the emissions from a firm in one country cause environmental damage in both countries. Hence, a given environmental target for each country can no longer be applied as a useful abstraction to make comparisons easier. In the model used in chapters 6 and 7, the governments balance environmental damage, valued by some damage function, with firm profits. If firms are large enough to have market power, they may also be large enough to have influence on environmental regulation. When a given environmental target is no longer assumed, interactions between firms and regulators must be included in the analysis.

Investment is considered to be investment in abatement capital in the chapters 6 and 7. That is, the role of capital as a substitute for the polluting input is made stronger than in chapters

[3] A mathematically precise definition of open-loop and feedback strategies was given in section 1.3.1.

[4] At the recent Kyoto meeting on global warming, it was discussed that countries with large forests should be given compensation in their emission reduction requirements.

2 and 3. This is done deliberately to analyse dynamic efficiency aspects in chapter 7. The strategic interaction that can occur in a model with strategic trade and transboundary pollution if all players (now two firms and two governments) use feedback strategies are quite complex. However, it is possible by taking derivatives, to sort out the effects of all interactions on investment and sign them for the special case of linear damage.

When firms and governments use open-loop strategies and governments balance environmental quality and firm profits, results for the model with transboundary pollution are different than under feedback strategies. In analogy with the results in chapter 2, it might be expected that with open-loop strategies, one instrument is clearly 'better'. However, that is not the case. To the contrary, the distortions of the instruments are more ambiguous. The reason is that for open-loop investment strategies a stricter environmental policy implies that investment in emission reduction becomes both more and less attractive. It becomes more attractive, because a stricter policy means an increase in the costs to be paid for emissions per unit of output. It becomes less attractive, since under stricter policy, the optimal output is reduced and hence emissions. In case of feedback strategies investment is neutral with respect to changes in current environmental policy, because investment in emission reduction is determined by its future marginal benefits. Under feedback strategies, the government is flexible in its policy choice, so that current policy has no direct link with future policy, which determines the marginal benefits from investment in emission reduction.

In chapter 4 the role of flexibility is stressed again, but in a somewhat different setting. This chapter models the investment behaviour of firms, in a closed economy, under a system of permits that gives firms some flexibility to allocate their emissions over time. Firms get this flexibility through transferable permits which they can bank or trade. The firms have more flexibility than under a strict emission standard, but they are less flexible than under environmental taxes. Their flexibility is further reduced, since they are required to satisfy individual emission standards at the end of the period of regulation. Since investment takes place over time, it is important for the firms to be flexible over time. That explains the cost savings, that may result even when firms are only allowed to reallocate their emissions individually, that is, can only bank permits for their own later use.

Bankable permits offer more flexibility than standards, but less than taxes. What may be expected then from these permits in an international duopoly? That is an interesting question for further research. With the ambiguities found in chapters 2, 3, 6 and 7 in mind and considering the complexity of a system of permits in case of investment, it is clear that it is not easy to answer this question. With permits, the government can no longer directly steer the amount of emissions at each point in time. In the models from chapters 2, 3, 6 and 7, that causes complications, since the objective of the government consists of flow damage (or a fixed flow target) and firm profits. When firms can bank permits it becomes a hard task for the government to balance these objectives. Firms namely need to know the path of future emission allowances to determine their investment. Through banking, they can reallocate their emissions over time and change the flow of emissions. Governments have to predict such reallocations and choose the optimal time path given their predictions. But if they cannot commit to such a path, time inconsistent solutions are probable. Firms invest based on their expectations about a certain path. Since the government also values firm profits it is intuitively attractive for the government to put some extra permits on the market once all firms have invested sufficiently.

3 Some last remarks

The chapters in this thesis all analyse environmental policy and international rivalry in compact and abstract models. The intention is to disentangle mechanisms, more than to find immediate policy implications. The result is a structured description of interactions that take place in a setting of oligopolistic international competition, transboundary pollution and governments that value both environmental quality and home-firm profits. With the use of dynamic game models, the role of decision strategies is explained and insight is gained into the concept of commitment which drives the results.

The research is conceptually rather than practically oriented. Yet, it is a nice exercise to dwell a little on possible policy implications. The four chapters on environmental policy competition use a differential game model that distinguishes firms, who produce to maximize profits from sales of output on an international market and governments, who regulate, to control some unidentified environmental problem. Possible applications are easy to find if the model is taken in a loose way. For example aluminium factories, large companies in the chemical sector and airports are well aware of their international competitors, while their production causes environmental pollution. Furthermore, the regulation aimed at these firms is often tailored to them and their international competitiveness is an issue for regulators.

It is not possible to directly translate the model to such applications. Application would require first to make adequate assumptions about the type of competition, the form of the environmental regulation and the type of decision strategies used by the players. The assumption of complete information and perfect foresight should be modified and it must be considered how players form their expectations and what information they can obtain at reasonable costs. Effects which were ignored to concentrate on the main issues, such as distortions in the environmental policy of other sectors, the possibility of entry, and income distribution effects should be considered to check whether their influence can indeed be safely ignored in the specific case at hand. It would not be surprising if in the end, the resulting model would be too complex to find analytical results. Numerical methods might then be useful.

In the presence of transboundary pollution and oligopolistic international trade, strategic effects play a role that cannot be ignored. This thesis shows that differential game models can provide important insights into this role. Commitment is the determinant behind all results. Different commitments related to taxes and standards explain why standards in some cases may reduce strategic overinvestment. Different commitments related to open-loop and feedback investment strategies explain why under feedback strategies the same result does not generally hold.

The model with transboundary pollution furthermore includes new results on dynamic efficiency. It sorts out all incentives to invest in emission reduction, in case of strategic trade, firms that interact strategically with governments, and governments who engage in environmental policy competition. An instrument is defined to be dynamically efficient if it comes close to the rate of investment in emission reduction that is socially optimal. If governments react to a higher environmental capital stock through stricter regulation, an incentive arises for firms to invest less in emission reduction. It is shown that cases exist where taxes provide too much incentives to invest, whereas under standards the firm comes closer to the socially optimal rate of investment.

In short, differential game models provide a rich structure that allows to consider various types of equilibria. Some of these equilibria, especially those that result when players are assumed to use flexible decision strategies, cannot be found in models with a less extensive dynamic structure. As a result, the analyses in this thesis detect mechanisms that were not (and could not) be found with the multistage models that are usually applied in the literature. Therefore, the analysis of differential game models adds to the understanding of the behaviour of oligopolistic firms and governments in case of strategic international trade and environmental policy competition.

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Samenvatting

Inleiding

Internationale aspecten zijn een kenmerk van veel milieuproblemen. Ze komen in verschillende vormen tot uitdrukking: De meest voor de hand liggende vorm is grensoverschrijdende vervuiling, zoals optreedt bij de lozing van giftige stoffen in de Maas of bij klimaatverandering. Meer indirect komen internationale aspecten tot uitdrukking in geval bedrijven lokale vervuiling veroorzaken, maar hun producten verhandelen op een internationale markt. Ook kunnen geïmporteerde producten tijdens of na hun consumptie voor vervuiling zorgen; een voorbeeld hiervan zijn batterijen. Dit proefschrift gaat hoofdzakelijk over de als tweede genoemde uitdrukkingsvorm: vervuilende producenten die te maken hebben met internationale concurrenten. Milieubeleid gericht op deze vervuilers zal hun (internationale) concurrentiepositie beïnvloeden. Immers, de concurrentie wordt gereguleerd door een andere overheid, met ander beleid. De overheid zal, in haar rol als ontwerper en uitvoerder van milieubeleid, rekening houden met de internationale concurrentiepositie van de bedrijven die aan dat beleid onderworpen worden, wanneer ze, naast een goede kwaliteit van het milieu, belang hecht aan de aanwezigheid van winstgevende bedrijven.

Strategische interactie tussen oligopolisten op de wereldmarkt kan leiden tot aanpassingen in milieubeleid. De verklaring daarvan kan worden gevonden door de interacties tussen bedrijven in oligopolie nader te bekijken. Een bedrijf dat concurreert op een markt met onvolledige mededinging kan profijt hebben van 'commitment', wat betekent 'het ergens aan gebonden zijn' (bijvoorbeeld aan milieuregulering). In een oligopolie zijn slechts enkele concurrenten op de markt aanwezig, waardoor de beslissingen van de één merkbare gevolgen hebben voor de ander. In evenwicht zal dan sprake zijn van strategisch gedrag: de bedrijven zullen rekening houden met de invloed die hun beslissingen hebben op hun concurrenten en met de reacties van die concurrenten. Het aangaan van commitments is een vorm van dergelijk strategisch gedrag: door zich te binden aan bepaalde acties, wordt de concurrent als het ware afgeschrikt. Het bedrijf kan bijvoorbeeld investeren in een grote productiecapaciteit en zich zo binden aan een hoge productie. Het wordt minder winstgevend voor de concurrent om zelf een hoge productie op de markt te brengen, omdat het grote aanbod op de markt dan tot een lage verkoopprijs kan leiden.

Het bedrijf kan zelf commitment aangaan, bijvoorbeeld door bepaalde investeringen te doen.

Daarnaast kan de overheid voor commitment zorgen, onder andere via milieubeleid. Milieuregulering vormt namelijk een commitment voor het bedrijf, wat de overheid door aanpassingen kan beïnvloeden. Zo kan ze haar positie als regelgever benutten en de oligopolist in het binnenland bevoordelen. Wanneer de overheid zowel hoge winst voor de oligopolist als een lage vervuiling nastreeft, zullen in evenwicht zulke aanpassingen optreden. Een centrale vraag in dit proefschrift is welk type instrument van milieubeleid de overheid het best kan toepassen, indien ze expliciet rekening houdt met internationale concurrentie en haar milieubeleid hieraan aanpast.

De vergelijking van diverse types milieubeleidsinstrumenten, zoals heffingen en maximum standaards op emissies, heeft een lange traditie in de milieu-economische literatuur (Zie Bohm en Russell, 1985). Een aantal verschillen tussen de diverse instrumenten behoort inmiddels tot de standaard literatuur. Pearce en Turner (1990) bijvoorbeeld noemen: verschillen in allocatieve efficiëntie, in effectiviteit, in dynamische efficiëntie en in transactiekosten. Een ander verschil heeft te maken met distributie-effecten: bij heffingen en in sommige systemen van verhandelbare emissierechten betalen de vervuilers een prijs voor elke eenheid emissie. Dat betekent een herverdeling van inkomsten tussen de vervuilers en de overheid. In de analyses in dit proefschrift wordt aangenomen dat de overheid de opbrengsten van heffingen of verhandelbare emissierechten kan terugsluizen naar de vervuilers, zodat dit effect verder buiten beschouwing kan worden gelaten.

Bovengenoemde verschillen betreffen de vergelijking van instrumenten in een gesloten economie, dat wil zeggen, er wordt geen rekening gehouden met onderlinge afhankelijkheden tussen landen, via handel of grensoverschrijdende vervuiling. Wanneer een open economie wordt beschouwd, moeten extra aspecten in de vergelijking worden betrokken. Zo kan vervuiling afkomstig uit een ander land niet altijd op een doeltreffende manier worden gereguleerd. Ook kunnen vervuilende bedrijven internationale concurrenten hebben. In geval van perfecte mededinging hebben de acties van individuele bedrijven geen merkbare invloed op elkaars mogelijkheden. Dan is het, ook in een open economie, optimaal voor de overheid om het milieubeleid te baseren op de gelijkheid van marginale kosten en marginale baten (zie Ulph (1994) en andere literatuur genoemd in hoofdstuk 1). Echter, in geval van competitie tussen een klein aantal concurrenten met marktmacht, is het aantrekkelijk voor de overheid om af te wijken van dat niveau van milieubeleid. Daarmee kan ze de binnenlandse bedrijven van concurrentievoordeel voorzien. Het verschijnsel dat de overheid haar beleid aanpast uit handelsstrategische overwegingen heet ook wel beleidsconcurrentie. Beleidsconcurrentie kan niet alleen plaatsvinden via milieubeleid, maar ook bijvoorbeeld via winstbelastingen of beleid op het gebied van de sociale zekerheid. Dit proefschrift onderzoekt de verschillen tussen milieubeleidsinstrumenten wanneer er sprake is van beleidsconcurrentie via het milieubeleid.

Ulph (1992) vergelijkt heffingen en standaards op emissie in een driestappenmodel van twee oligopolisten die ieder in een ander land gevestigd zijn. In een eerste fase beslist de overheid welk instrument ze inzet, heffingen of standaards, om de vervuiling tot een bepaald maximum te beperken. In een tweede fase kiest het bedrijf de hoeveelheid kapitaalgoederen die in de productie wordt gebruikt. Tenslotte kiest het bedrijf de omvang van de productie en daarmee de vervuiling. De keuze voor heffingen dan wel standaards beïnvloedt de strategische interactie tussen de bedrijven. Ulph concludeert dat in evenwicht beide overheden voor standaards kiezen, omdat het eigen bedrijf dan hogere winsten kan behalen, terwijl de vervuiling even groot is als onder (goed gekozen) heffingen. Het is opvallend dat standaards tot hogere winsten leiden, zelfs als wordt gecorrigeerd voor de betaling van de heffingen. Als er geen sprake zou zijn van strategische interactie tussen de duopolisten, dan zouden de bedrijven, afgezien van de betaling voor de heffingen, dezelfde winst halen onder beide instrumenten. Door de strategische interactie tussen de bedrijven is er sprake van overinvesteringen, welke verschillend zijn onder heffingen en standaards.

Het verschil tussen heffingen en standaards wordt dus verklaard via hun invloed op de keuze van de kapitaalgoederenvoorraad. De keuze voor een bepaalde kapitaalgoederenvoorraad is echter geen eenmalige keuze. Kapitaal wordt opgebouwd via investeringen. Investeringen in en productie met kapitaalgoederen vinden plaats over de tijd. Investeringen hebben daarom niet alleen effect op de winst van het bedrijf op het tijdstip waarop ze plaatsvinden maar ook op de winst in de periode daarna. Interacties tussen de betrokken economische agenten, overheden en bedrijven vinden ook over een langere periode plaats. De mate van reactie op elkaars gedrag hangt af van de flexibiliteit van de betrokkenen en van de gekozen beslissingsstrategieën.

Dit is aanleiding om in dit proefschrift de dynamiek van de interactie tussen de bedrijven en tussen de overheden nader uit te werken. In plaats van een stappenmodel wordt de interactie tussen de bedrijven en de overheden als een differentiaalspel gemodelleerd. Differentiaalspelen maken het mogelijk om het dynamisch proces van interacties gedetailleerd te modelleren. Hierdoor kan interactie tussen bedrijven en overheden over de tijd geanalyseerd worden. Dat geeft inzicht in de effecten van verschillende beslissingsstrategieën en maakt het mogelijk om investeringen in kapitaal expliciet als proces in de tijd te bestuderen.

Inhoud van het proefschrift

In hoofdstuk 1 staat ter inleiding een overzicht van relevante literatuur op het gebied van strategische internationale handel en differentiaalspelen. Eerder onderzoek naar de relatie tussen handel en milieu wordt kort behandeld, met de nadruk op zogenaamde 'stappenmodellen' van internationale oligopolies. Ook geeft dit hoofdstuk een introductie op de differentiaalspelen die in de rest van het proefschrift worden gebruikt.

De hoofdstukken 2 en 3 analyseren een duopolie, dat wil zeggen een tweelandenmodel met één bedrijf per land. Door deze abstractie verdwijnen de verschillen in allocatieve efficiëntie tussen heffingen en standaards. Bij aanname vindt consumptie van de output van het duopolie volledig plaats in een derde land. Bovendien wordt geabstraheerd van verschillen in transactiekosten, in doeltreffendheid en in verdelingseffecten tussen heffingen en standaards. De nadruk komt zo te liggen op verstoringen uit handelsstrategische overwegingen en op dynamische efficiëntie. De hoofdstukken 2 en 3 vergelijken heffingen met standaards voor respectievelijk investeringsstrategieën met weinig flexibiliteit (open-loop strategieën) en investeringsstrategieën met veel flexibiliteit (feedback strategieën).

Een derde instrument van milieubeleid, verhandelbare emissierechten wordt geanalyseerd in

hoofdstuk 4. In dit hoofdstuk analyseren we het investeringsgedrag van een bedrijf in een gesloten economie onder een systeem van emissierechten die verhandelbaar zijn en op de 'bank' kunnen worden gezet voor toekomstig gebruik. Bedrijven hebben zo enige flexibiliteit om hun emissies over de tijd te verdelen. Verder stelt het model in dit hoofdstuk eisen aan de kapitaalgoederenvoorraad van de bedrijven aan het eind van een bepaalde periode.
Grensoverschrijdende vervuiling en de dynamische interactie tussen landen is onderwerp van hoofdstuk 5. Er wordt geabstraheerd van bedrijven en overheden. Dit hoofdstuk gaat over de rol van productiebossen bij de vermindering van de uitstoot van kooldioxide naar de atmosfeer. Groeiende bomen assimileren kooldioxide. Een land met grote productiebossen, zoals bijvoorbeeld Finland, kan redeneren dat het via de groei van zijn bossen bijdraagt aan bestrijding van klimaatverandering. Zulke landen pleiten ervoor om te onderhandelen over de netto emissies van kooldioxide, dat wil zeggen de emissies waarin assimilatie door bossen wordt meegeteld. In dit hoofdstuk leiden we evenwichten af waarbij zulke netto emissies worden gebruikt in plaats van de bruto emissies van kooldioxide.
De hoofdstukken 6 en 7 borduren voort op hoofdstuk 2 en 3. De analyse wordt uitgebreid naar grensoverschrijdende vervuiling. Grensoverschrijdende vervuiling is op zich een reden voor de overheid om af te wijken van het milieubeleid wat optimaal is voor een gesloten economie. De overheid zal immers niet altijd rekening houden met emissies van het eigen bedrijf die in het buitenland voor vervuiling zorgen en kan op haar beurt geen controle uitoefenen op vervuiling afkomstig van bedrijven over de grens. Bovendien zal de verstoring in het milieubeleid voor handelsstrategische doeleinden versterkt worden.
In de modellen in deze hoofdstukken maakt de overheid een afweging tussen milieukwaliteit in het binnenland en bedrijfswinsten van het eigen bedrijf. Naast interactie tussen bedrijven en interactie tussen overheden, wordt ook de interactie tussen bedrijven en overheden geanalyseerd. Investeringen worden specifiek als investeringen in de vermindering van emissies beschouwd. Dat wil zeggen, de bedrijven investeren in zogeheten milieutechnologie. Kapitaal is daarmee, meer nog dan in hoofdstukken 2 en 3, een substituut voor emissies. Hoofdstuk 6 analyseert de verstoringen in milieubeleid voor heffingen en standaards. Hoofdstuk 7 gaat expliciet in op dynamische efficiëntie en analyseert in hoeverre de investeringen in emissiereductie onder heffingen en standaards optimaal zijn, wanneer de overheid het milieubeleid aanpast uit handelsstrategische overwegingen.
Hoofdstuk 8 tenslotte sluit af met een samenvatting van de resultaten.

Samenvatting van de resultaten

In deze paragraaf worden de belangrijkste resultaten uit het proefschrift uiteengezet. Tussen de hoofdstukken 2, 3, 6 en 7 is een duidelijk verband, omdat ze een vergelijkbaar model analyseren. Daarom zal de nadruk eerst op de resultaten uit die hoofdstukken liggen. Aan het eind komen ook de resultaten van hoofdstuk 4 en 5 aan de orde.
Uit bovengenoemde vier hoofdstukken kunnen de volgende twee algemene conclusies worden getrokken: Ten eerste verschillen heffingen en standaards inderdaad wanneer de overheid

handelspolitieke overwegingen meeweegt in haar milieubeleid. Ten tweede is op grond van deze verschillen niet één instrument als 'beter' dan het andere instrument aan te wijzen. Welk instrument 'beter' is vanuit de optiek van een overheid die milieu en bedrijfswinst beide meetelt, is onder meer afhankelijk van het type beslissingsstrategie dat door bedrijven wordt gebruikt bij hun investeringen. Deze twee conclusies zijn te verklaren door nader in te gaan op verschillen in commitment. Hiermee kan worden uitgelegd waarom een ander evenwicht tot stand komt bij een andere investeringsstrategie of een ander type milieubeleidsinstrument.

De overheid kan het eigen bedrijf bevoordelen door de strategische aanpassing van het milieubeleid dankzij het commitment wat de overheid aan dat bedrijf kan geven. Wanneer de bedrijven hun marktmacht uitoefenen via keuze van de omvang van de productie, is het een voordeel voor het bedrijf om een commitment te hebben om veel te produceren. De overheid kan zulke commitments direct aan het bedrijf geven, bijvoorbeeld in de vorm van exportsubsidies, maar ook indirect, via verstoringen in het milieubeleid (een lagere heffing of een minder strenge standaard). Mits juist gekozen, resulteren zulke verstoringen in meer winst voor het bedrijf. De overheid moet deze extra winst afwegen tegen het verlies in milieukwaliteit als gevolg van het minder strenge milieubeleid. Het kan aangetoond worden dat een evenwicht bestaat waarin beide overheden een kleine verstoring hebben in hun milieubeleid (zie bijvoorbeeld Ulph, 1994). Omdat de overheid in elk van de twee landen op dezelfde manier zal redeneren ontstaat een evenwicht waarbij alle partijen strategisch handelen.

Heffingen en standaards als instrumenten van milieubeleid verschillen in de mate van commitment die ze voor het bedrijf impliceren. Een standaard op de emissies betekent een sterk commitment voor wat betreft het gebruik van de vervuilende productiefactor. In evenwicht zal het bedrijf juist de toegestane maximum emissie uitstoten. Het is dan niet optimaal om de emissies aan te passen voor kleine veranderingen in de acties van de concurrent. Onder een heffing daarentegen is het bedrijf flexibel in de keuze van de hoeveelheid vervuilende input die het gebruikt.

Met een differentiaalspel is het mogelijk om de invloed van de investeringsstrategie te analyseren. De investeringsstrategie bepaalt de mate van commitment met betrekking tot de productiefactor kapitaal en beïnvloedt daardoor het commitment op output. Beslissingsstrategieën waarbij het bedrijf reageert op de huidige situatie, feedback strategieën, impliceren flexibele reactie en daarmee weinig commitment. Strategieën waarbij het bedrijf een investeringspad volgt zonder te reageren op de huidige situatie, open-loop strategieën, impliceren minder flexibiliteit en meer commitment.

Met een open-loop investeringsstrategie wordt het tijdpad van de investeringen in feite aan het begin van het spel vastgelegd. Dan zal de overheid standaards prefereren boven heffingen omdat standaards bij dezelfde emissies tot hogere bedrijfswinsten leiden. Het door de overheid opgelegde commitment van het bedrijf op emissies impliceert namelijk een commitment op output. Daardoor is het minder zinvol voor bedrijven om strategische investeringen te doen. Het gevolg is een evenwicht met minder overinvesteringen en hogere winsten dan het vergelijkbare evenwicht onder heffingen. Dit resultaat wordt afgeleid in hoofdstuk 2 en bevestigt eerdere resultaten van Ulph in een meerstapsmodel die genoemd zijn in paragraaf 3 hierboven.

In hoofdstuk 3 wordt aangetoond dat dit resultaat afhankelijk is van de veronderstelde investeringsstrategie, namelijk een open-loop strategie. Bij feedback investeringsstrategieën, waarbij investeringen worden aangepast aan de huidige situatie, is het niet langer mogelijk om één instrument aan te wijzen als 'beter'. Heffingen zijn onder sommige omstandigheden beter dan standaards. In geval van feedback investeringen kunnen bedrijven elkaars investeringsbeslissingen strategisch beïnvloeden. Een numeriek voorbeeld laat zien dat het dan afhankelijk is van specifieke kenmerken van de bedrijven welk instrument beter is. In het voorbeeld zullen bij een lage rente, kleine depreciatie en hoge investeringskosten heffingen beter zijn dan standaards.

In de hoofdstukken 6 en 7 wordt grensoverschrijdende vervuiling toegevoegd. Naast interactie tussen bedrijven en tussen overheden, is ook interactie tussen overheid en bedrijf geanalyseerd. Wanneer zowel overheden als bedrijven feedback strategieën gebruiken zijn veel interacties mogelijk en ontstaan zeer complexe evenwichten. Voor een specifiek geval kunnen alle effecten op de investeringen in het evenwicht nader toegelicht worden.

Opnieuw zijn in geval van open-loop investeringsstrategieën de resultaten anders dan in geval van feedback strategieën. Een voor de hand liggende gedachte is dat, analoog aan de resultaten in hoofdstuk 2, onder open-loop investeringsstrategieën één instrument aan te wijzen is dat 'beter' is. Dit blijkt niet het geval te zijn. Dat komt omdat het effect van een strenger milieubeleid op de incentives voor het bedrijf om in reductie van de emissies te investeren onder open-loop strategieën ambigu is. Investeren in reductie wordt aantrekkelijker, omdat de toekomstige opbrengsten van de investering, het vermijden van de (hogere) kosten per eenheid emissie, als gevolg van een strenger milieubeleid toenemen. Aan de andere kant wordt investeren minder aantrekkelijk, omdat de optimale hoeveelheid productie daalt en daarmee ook de totale hoeveelheid emissies. In geval van feedback investeringsstrategieën en feedback overheidsbeleid hebben verstoringen in het huidige milieubeleid geen invloed op de investeringen, omdat de overheid in dat geval flexibel is in haar beleidskeuze en het huidige beleid geen effect heeft op het toekomstige beleid (wat de winstgevendheid van investeringen bepaalt). Daardoor is het bij feedback investeringen wel mogelijk om voor een vrij algemene specificatie te bepalen in welke richting de totale verstoring van het milieubeleid zal gaan in de diverse scenario's die in hoofdstuk 6 worden geanalyseerd.

De analyse in hoofdstuk 7 concentreert zich op de dynamische efficiëntie van heffingen en standaards. Een instrument van milieubeleid is meer dynamisch efficiënt genoemd naarmate de investeringen dichter liggen bij de investeringen die optimaal zijn vanuit het standpunt van een planner in ieder land die waarde hecht aan milieukwaliteit en bedrijfswinst over de hele periode. Het is mogelijk om een voorbeeld te vinden waarbij onder heffingen in evenwicht 'teveel' wordt geïnvesteerd en de investeringen onder standaards dichter bij het sociale optimum komen. Dit is voornamelijk het gevolg van de strategische interactie tussen overheden en bedrijven die bij heffingen in dat geval de vorm aanneemt van extra investeringen met als doel een lagere heffing.

Tot slot van deze paragraaf komen de resultaten in de overige twee hoofdstukken aan de orde. In hoofdstuk 4 vinden we dat bedrijven, die investeren in emissiereductie en te maken hebben met een systeem van verhandelbare emissierechten, profiteren van de mogelijkheid om hun emissierechten op de bank te kunnen zetten voor later gebruik en hun investeringen over de tijd hieraan aanpassen. Omdat investeringen een proces in de tijd zijn, is het van belang voor bedrijven om enige flexibiliteit over de tijd te hebben. Dat verklaart de kostenbesparingen die bedrijven kunnen behalen, zelfs in een systeem waarin ze hun emissierechten niet kunnen verhandelen maar alleen op de bank kunnen zetten. Verhandelbare emissierechten bieden meer flexibiliteit dan standaards, maar minder dan heffingen. Het is interessant om te overwegen wat het resultaat zal zijn van dit instrument van milieubeleid in de setting van de eerder besproken hoofdstukken, waarbij de overheid het beleid verstoort uit handelspolitieke overwegingen. Gegeven de complexiteit van de analyses in de hoofdstukken 2, 3, 6 en 7, is het duidelijk dat deze vraag niet zomaar te beantwoorden is. Onder dergelijke emissierechten heeft de overheid geen directe controle meer op de emissies van het eigen bedrijf op een bepaald tijdstip, want het bedrijf beïnvloedt de allocatie van emissies over de tijd als het een permit op de bank zet. Dat betekent dat het model aangepast moet worden, omdat eerder uitgegaan werd van ofwel gegeven emissies op elk tijdstip (in de hoofdstukken 2 en 3) ofwel een doelstelling waarin de emissies op ieder tijdstip tellen.

Hoofdstuk 5 geeft analyses van evenwichten tussen een land zonder en een land met bos, die over hun emissies van kooldioxide moeten beslissen. Het land zonder bos profiteert eveneens als bos wordt meegeteld bij de bepaling van de netto emissies van kooldioxide van de landen. Ook dit land kan namelijk rekenen op de extra mogelijkheid tot assimilatie van kooldioxide en kan daardoor een hogere emissie kiezen terwijl de uiteindelijke accumulatie van kooldioxide in de atmosfeer lager uitvalt. Het meetellen van kooldioxide-assimilatie door bossen betekent een verandering in de benutting van het bos. Voor een bos wat om te beginnen redelijk groot is zal de oogst van bomen lager zijn wanneer bossen wel meetellen bij bepaling van de (netto) emissie van kooldioxide dan wanneer dat niet het geval is.

Tenslotte

Alle modellen in dit proefschrift zijn conceptueel van aard. Ze zijn opgezet om na te denken over mechanismen en deze systematisch te analyseren. Het is uitdrukkelijk niet de bedoeling geweest iets te zeggen over concreet beleid. Dat zou een ander type modellen vereisen, met een empirische invulling en bijgevolg meer details en aandacht voor eventuele indirecte effecten. De analyses onderzoeken op systematische manier de interacties die kunnen plaatsvinden wanneer de overheid handelspolitieke overwegingen laat meetellen in haar milieubeleid, in het geval dat dit beleid gericht is op enkele grote vervuilers met marktmacht. Met behulp van dynamische speltheorie worden de effecten van diverse aannames met betrekking tot de beslissingsstrategieën uitgewerkt.

Het begrip 'commitment' blijkt een centraal begrip om de interacties te verklaren. Verschillen in commitment tussen heffingen en standaards verklaren waarom in hoofdstuk 2 gevonden wordt dat standaards een 'beter' instrument van milieubeleid zijn dan heffingen. Verschillen in commitment als gevolg van open-loop dan wel feedback investeringsstrategieën verklaren waarom dat resultaat alleen geldt bij open-loop strategieën. Bij feedback strategieën kan het omgekeerde, waarbij heffingen 'beter' zijn dan standaards, ook voorkomen.

Differentiaalspelen vormen daarmee een rijke modelstructuur, waarin verschillende typen evenwichten bekeken kunnen worden. Sommige van deze evenwichten kunnen niet worden geanlyseerd in simpeler modellen, waarbij de dynamische structuur minder expliciet gemodelleerd wordt. Dat zijn die evenwichten waarbij de spelers gebruik maken van flexibele beslissingsstrategieën en hun beslissingen aanpassen aan de toestand op het moment. Indien de spelers zulke flexibele strategieën gebruiken, doen zich interacties voor die niet gemodelleerd kunnen worden met behulp van de meerstapsmodellen die gebruikelijk zijn in de literatuur van strategische internationale handel en beleidsconcurrentie. Differentiaalspelen modelleren gedrag wat zich afspeelt over de tijd, zoals de accumulatie van kapitaal, heel precies. Dat is belangrijk om het gedrag van bedrijven en overheden in een situatie van strategische internationale handel met milieubeleid te begrijpen.

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Stellingen

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Environmental policy instruments and international rivalry

Talitha Feenstra

1. Een overheid die milieubeleid ontwerpt voor een bedrijf wat concurreert op een internationale markt met onvolledige mededinging en belang hecht zowel aan goede milieukwaliteit als aan winst voor het bedrijf zal bij de keuze van het instrument van milieubeleid rekening houden met de mate van commitment die het betreffende instrument verschaft. (hoofdstukken 2 en 3)

2. De accumulatie van kapitaal is een dynamisch proces. Een analyse waarbij investeringsbeslissingen worden teruggebracht tot een eenmalige keuze van de kapitaalgoederenvoorraad kan daarom tot verkeerde conclusies leiden. (hoofdstukken 2 en 3)

3. Een "bank for permits" dat wil zeggen de mogelijkheid voor bedrijven om emissiequota te heralloceren over de tijd, leidt tot besparingen indien de kosten van emissiereducties voornamelijk voortvloeien uit investeringen en daarmee verband houdende aanpassingskosten. (hoofdstuk 4)

4. Wanneer expliciet rekening wordt gehouden met de kooldioxide-opname van bossen, zal gekozen worden voor minder kap en een groter bos. Dit geldt voor een situatie waarin het bos initieel al redelijk groot is en gaat gepaard met een lagere reductie in energiegebruik dan wanneer de kooldioxidedynamiek van bossen wordt genegeerd. (hoofdstuk 5)

5. Wat complex lijkt is soms (iets) simpeler dan het schijnbaar eenvoudige. (hoofdstuk 6)

6. Als in de situatie beschreven in stelling 1 sprake is van grensoverschrijdende vervuiling, is voor wat betreft hun dynamische efficiëntie geen algemene uitspraak te doen over emissieheffingen en -standaards. Geen van beide is beter dan de ander in dit opzicht. (hoofdstuk 7)

7. Het succes van het station als vestigingsplaats voor middenstand is vooral te danken aan slechte dan wel gemiste aansluitingen.

8. Wanneer het doel het scheppen van werkgelegenheid is, is beleidsconcurrentie tussen lagere overheden om bedrijven tot vestiging binnen de gemeentegrenzen te verleiden, gegeven de korte reisafstanden in Nederland, weinig zinvol.

9. "Onthaasten" veronderstelt een dermate bewuste omgang met tijd dat het daarmee onmogelijk wordt.

TALITHA FEENSTRA g[illegible] Groningen in 1993. She carried out her PhD research in the field of environmental economics at Tilburg University, for the Netherlands Organisation for Scientific Research (NWO). In 1998, she started working in the field of health economics, on a joint research project between the Erasmus University Rotterdam and the National Institute of Public Health and the Environment (RIVM).

The effects on the environment of international interdependencies between countries are manifold. One concern is the occurrence of environmental policy competition, or the distortion of environmental policy measures for strategic trade reasons. This thesis analyses environmental regulation of polluting firms which sell their output on an international market characterised by imperfect competition. Governments value environmental quality as well as profits for their domestic oligopolists and behave strategically. The analysis compares environmental taxes with standards on emissions. Differential game models are used to accurately model the accumulation of capital through investment. Equilibria with feedback investment strategies lead to ambiguity in the choice of instruments which can not be found in the multistage models common in the literature on strategic trade. The investment behaviour of a firm subject to a system of permits that are transferable over time is the subject of the fourth chapter. Chapter five is concerned with carbondioxide sequestration by forests. In the last two chapters, transboundary pollution and trade are combined to analyse the distortions of environmental policy instruments and firms' investment in environmental capital.

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